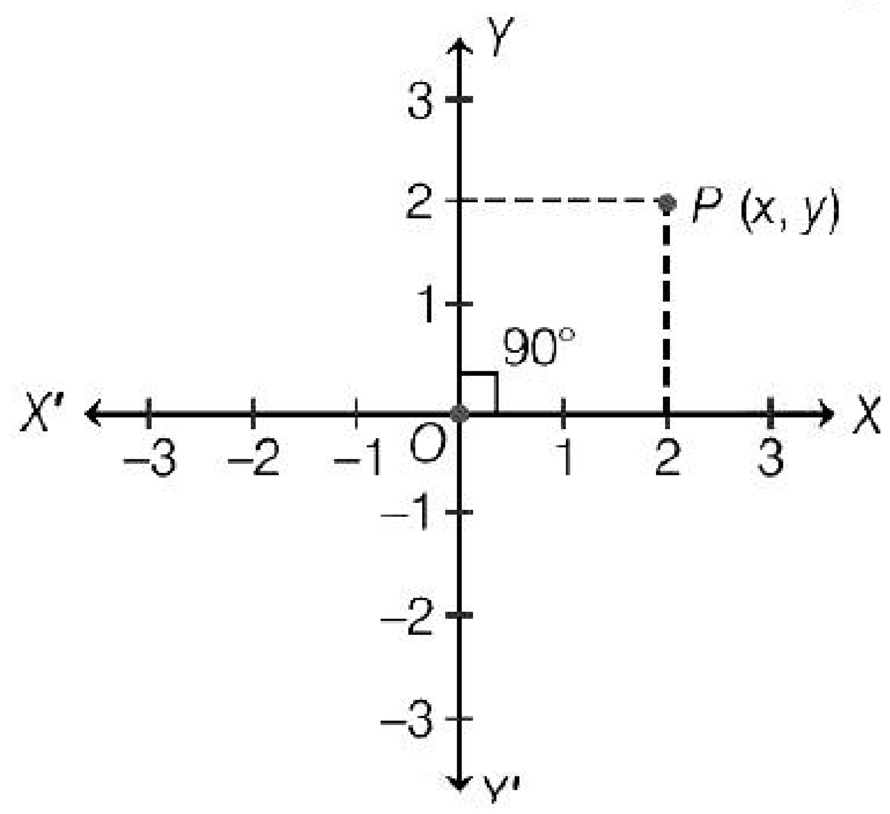
Coordinate Geometry

Quick Revision

Cartesian System

The system used to describe the position of a point in a plane, is called cartesian system. In cartesian system, there are two mutually perpendicular straight lines XX' and YY', which intersect each other at origin point O.



The horizontal line XOX' is called X-axis (or abscissa) and the vertical line YOY' is called Y-axis (or ordinate).

Distance between Two Points in a Cartesian Plane

The distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or $$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

i.e. $$PQ = \sqrt{(\text{Difference of abscissae})^2 + (\text{Difference of ordinates})^2}$$

If one coordinate is at origin say $P(x_1, y_1) = P(0, 0)$, then distance between two points is

$$PQ = \sqrt{x_2^2 + y_2^2}$$

Collinear Points

When three or more than three points lie on a same line, then they are called collinear points.

Suppose A, B and C are three points, then the condition for collinearity of three points is

$$AB + BC = AC$$

or $$AC + CB = AB$$

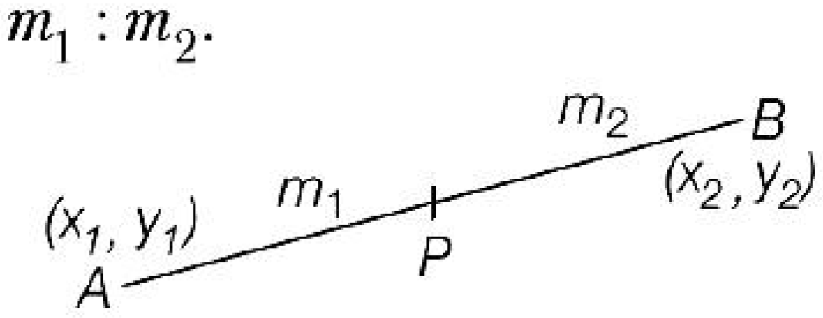
or $$BA + AC = BC$$

Section Formulae

In section formula, we find the coordinates of a point which divides the given line segment internally (or externally) in a given ratio.

Internal Division of a Line Segment

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points and $P(x, y)$ is a point on the line segment joining A and B such that $AP : BP = m_1 : m_2$, then point P is said to divide line segment AB internally in the ratio $m_1 : m_2$.



The coordinates of point P are given by

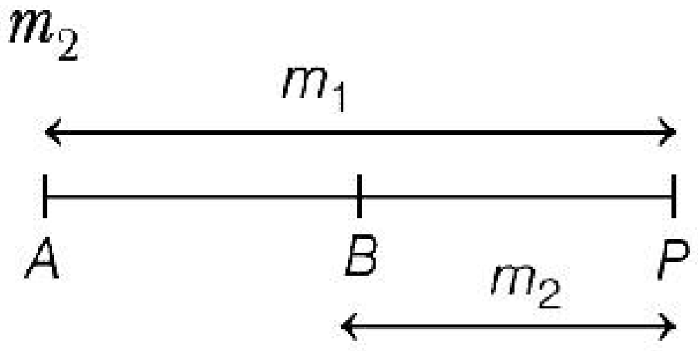
$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

Generally, for finding internal division ratio, we consider P divides AB in the ratio $k : 1$, then the coordinates of the point P will be

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right).$$

External Division of a Line Segment

If a point P divides line segment AB externally in the ratio $m_1 : m_2$



Then, coordinates of point P are

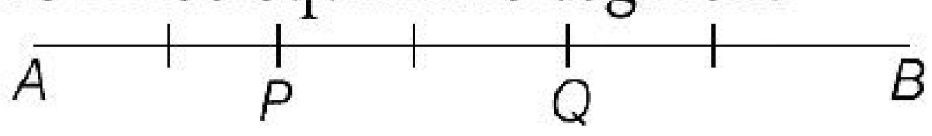
$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

This formula is known as section formula for external division.

Coordinates of Mid-point of Line Segment

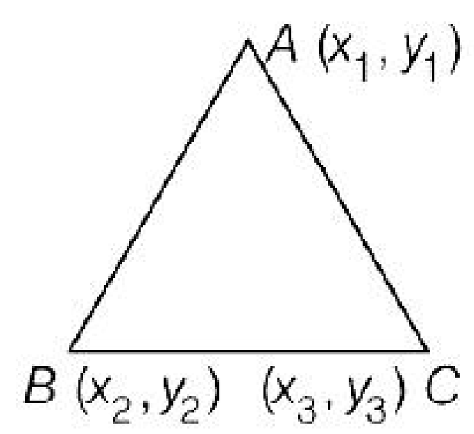
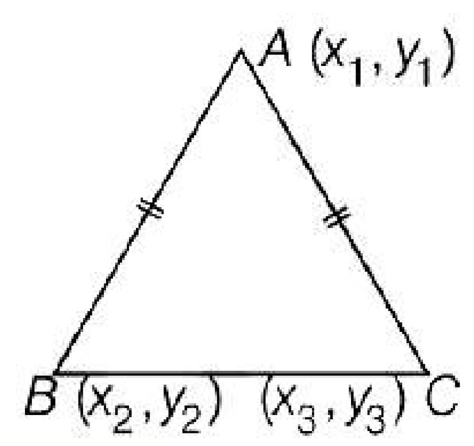
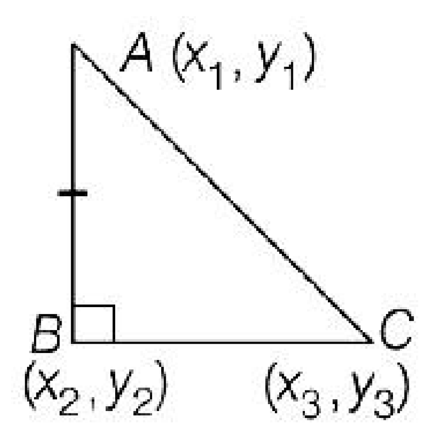
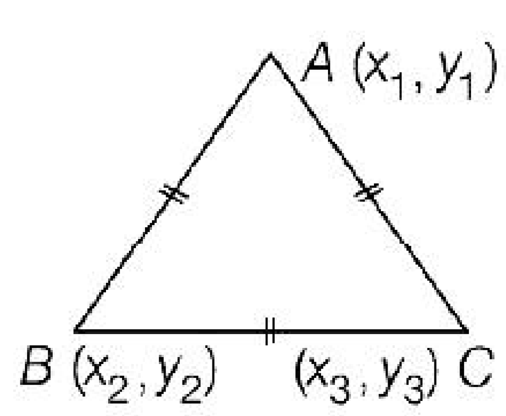
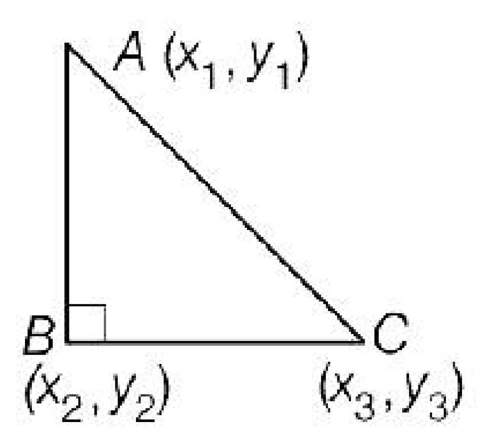
If the point P divides the line segment equally i.e., 1 : 1, then the coordinates of P will be $\left(\frac{x_1 + y_1}{2}, \frac{y_1 + y_2}{2}\right)$. This is also called **mid-point formula**.

Note Trisection of the line segment means, a line is divided into three equal line segment

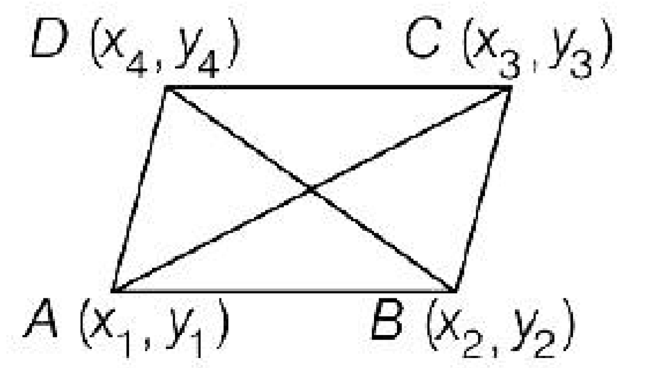
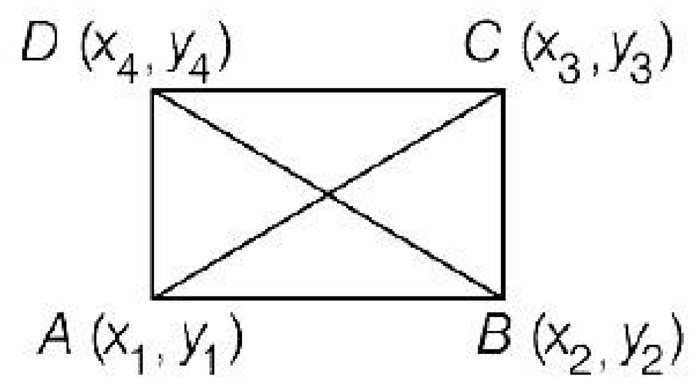
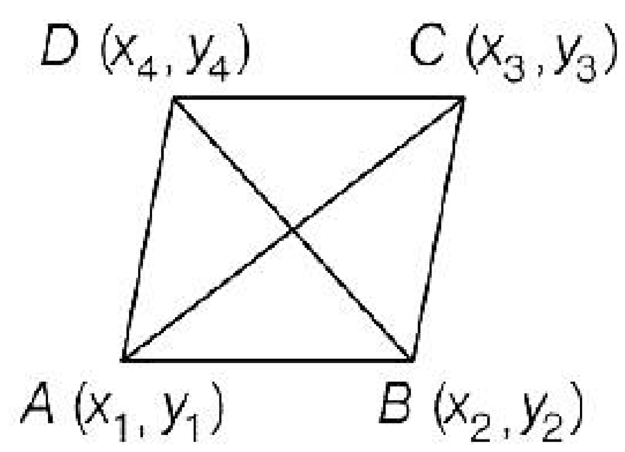
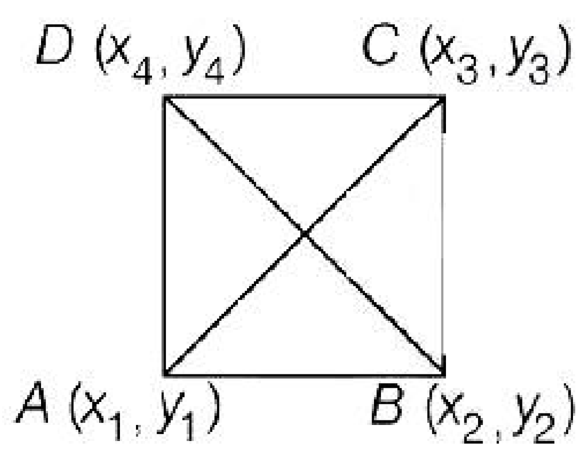


i.e. $AP = PQ = QB$.

Different Types of Triangle and Their Conditions

Types of Triangle	Figures	Properties	Conditions
Scalene triangle		All sides are unequal.	$AB \neq BC \neq CA$
Isosceles triangle		Length of any two sides are equal.	$AB = AC \Rightarrow \angle C = \angle B$
Right isosceles triangle		Angle between two equal sides is 90°.	$AB = BC$ and $\angle B = 90°$
Equilateral triangle		Length of all the three sides are equal.	$AB = BC = CA$ and $\angle A = \angle B = \angle C = 60°$
Right angled triangle		Sum of the squares of two sides is equal to the square of third side. It is also called Pythagoras theorem.	$AC^2 = AB^2 + BC^2$

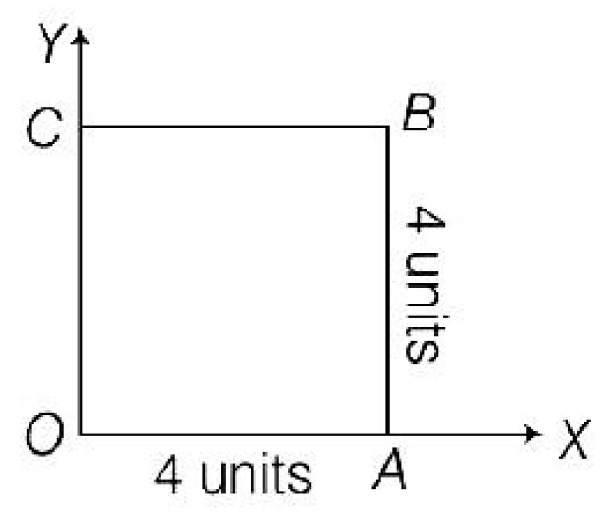
Different Types of Quadrilateral and Their Conditions

Types of Quadrilateral	Figures	Properties	Conditions
Parallelogram		Both pairs of opposite sides are equal. Diagonals bisect each other.	$AB = CD$, $AD = BC$ and $AC \neq BD$
Rectangle		A parallelogram with equal diagonals. Diagonals bisect each other.	$AB = CD$, $AD = BC$ and diagonal AC = diagonal BD
Rhombus		A parallelogram with all the sides are equal. Diagonals bisect each other at right angle.	$AB = BC = CD = DA$ and $AC \neq BD$
Square		A rhombus with equal diagonals. Diagonals bisect each other at right angle.	$AB = BC = CD = DA$ and diagonal AC = diagonal BD

Objective Questions

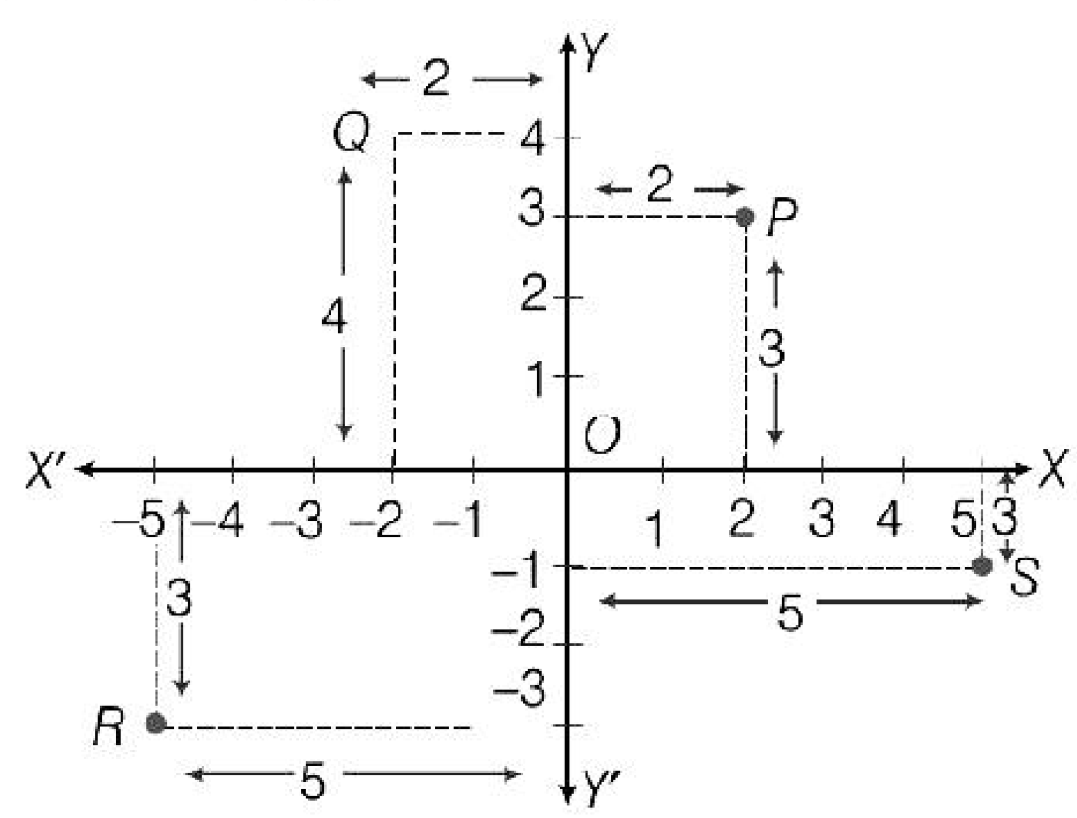
Multiple Choice Questions

1. In the given figure, O is the intersecting point of OA and OC and $OABC$ is a square of side 4 units, then the position of A, B and C is



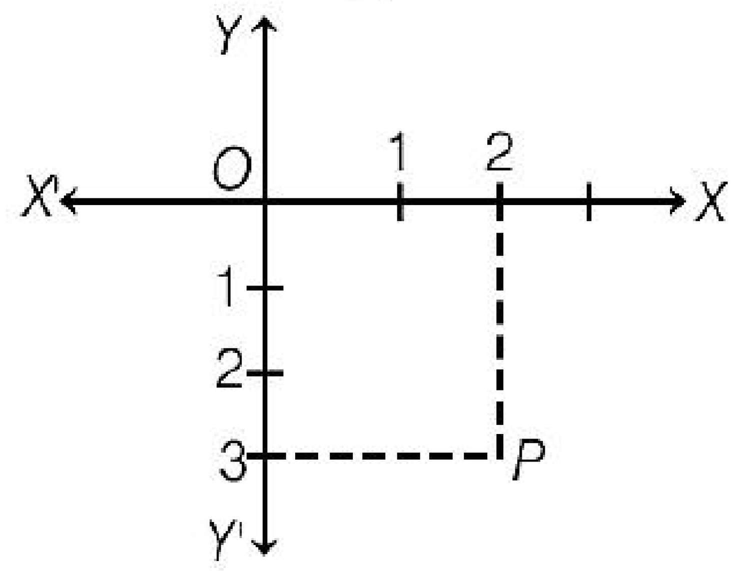
(a) (4, 0)(4, 4)(0, 4)
(b) (4, 0)(0, 4)(4, 4)
(c) (0, 4)(4, 4)(4, 0)
(d) None of the above

2. In the given figure, the ordinates of the points P, Q, R and S is



(a) 2, −2, −5, 5
(b) 3, 4, −3, −1
(c) 3, 4, −5, 5
(d) 2, 4, −5, −1

3. The coordinates of the point P as shown in the diagram will be



(a) $(2, -3)$ (b) $(-3, 2)$
(c) $(2, 3)$ (d) $(3, 2)$

4. The coordinate of the vertices of a rectangle whose length and breadth are 6 and 4 units, respectively. Its one vertex is at the origin. The longer side is on the X-axis and one of the vertices lies in second quadrant is

(a) $(0, 0)(6, 4)(6, 0)(0, 4)$
(b) $(0, 0)(0, 4)(6, 0)(6, 4)$
(c) $(0, 0)(6, 4)(-6, 0)(6, 4)$
(d) $(0, 0)(0, 4)(-6, 4)(-6, 0)$

5. The point $P(-4, 2)$ lies on the line segment joining the points $A(-4, 6)$ and $B(-4, -6)$.

(a) True (b) False
(c) Can't say (d) Partially True/False

6. The distance of the point $P(2, 3)$ from the X-axis is ***[NCERT Exemplar]***

(a) 2 units (b) 3 units
(c) 1 units (d) 5 units

7. The distance between the points $P(-6, 7)$ and $Q(-1, -5)$ is

(a) -6 units (b) 13 units
(c) 1 units (d) 5 units

8. The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$, is

(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

9. The value of y, if the distance between the points $(2, y)$ and $(-4, 3)$ is 10 is

(a) 6 (b) -11 (c) 5 (d) 11

10. Point $P(0, 2)$ is the point of intersection of Y-axis and perpendicular bisector of line segment joining the points $A(-1, 1)$ and $B(3, 3)$.

(a) True (b) False
(c) Can'tsay (d) Partially True/False

11. If the distance between the points $(4, p)$ and $(1, 0)$ is 5, then the value of p is

(a) 4 (b) -4
(c) Both (a) and (b) (d) 0

12. A circle has its centre at the origin and a point $P(5, 0)$ lies on it. The point $Q(6, 8)$ lies outside the circle.

(a) True (b) False
(c) Can't say (d) Partially True/False

13. The radius of the circles whose centre is at $(0, 0)$ and which passes through the points $(-6, 8)$ is ....

(a) 10 units (b) 11 units
(c) 9 units (d) 8 units

14. Is the points $(1, -1)$, $(5, 2)$ and $(9, 5)$ are collinear?

(a) Yes
(b) No
(c) Can't find
(d) None of the above

15. If the point $P(2, 1)$ lies on the line segment joining points $A(4, 2)$ and $B(8, 4)$, then ....

(a) $AP = \frac{1}{3}AB$ (b) $AP = PB$
(c) $PB = \frac{1}{3}AB$ (d) $AP = \frac{1}{2}AB$

16. If the point $P(x, y)$ is equidistant from the points $A(5, 1)$ and $B(1, 5)$, then

(a) $y = 3x$ (b) $x = y$
(c) $x = -8y$ (d) $-8x = y$

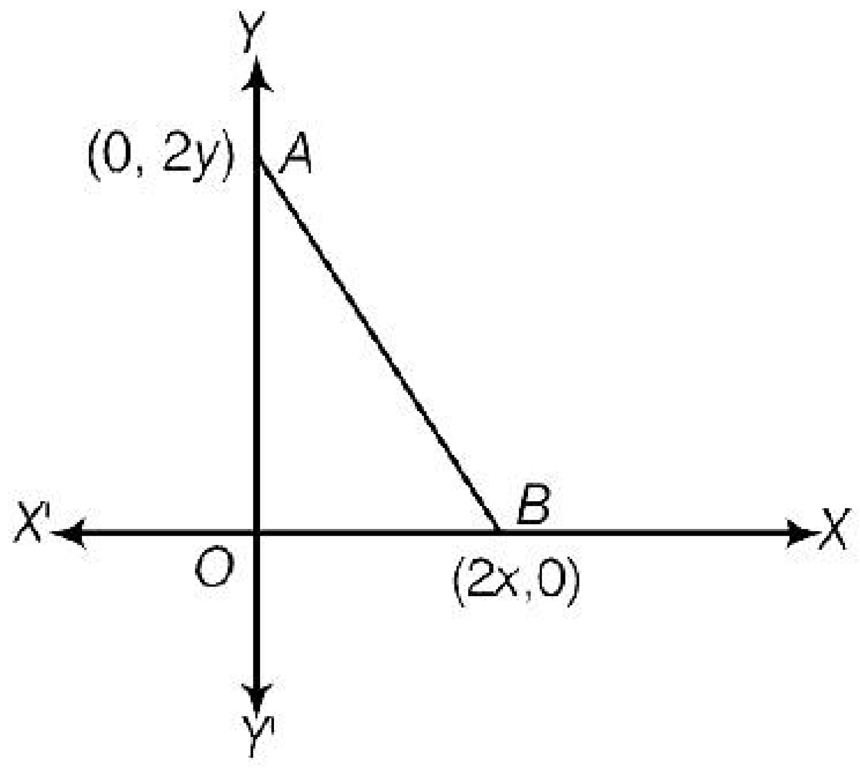
17. A point on X-axis which is equidistant from the points $(1, 3)$ and $(-1, 2)$.

(a) $(5/2, 0)$ (b) $(5, 0)$
(c) $(4, 0)$ (d) $(5/4, 0)$

18. The point on X-axis, which is equidistant from the point (7, 6) and (−3, 4) is

(a) (0, 3) (b) (4, 3)
(c) (3, 0) (d) None of these

19. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is



(a) (x, y) (b) (y, x)
(c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$

20. The coordinate of a point on Y-axis which is equidistant from the point $A(6, 5)$ and $B(-4, 3)$, will be

(a) (0, 9) (b) (0, −9)
(c) (0, 5) (d) (0, 3)

21. The radius of the circle whose end points of diameter are (24, 1) and (2, 23) is

(a) $22\sqrt{2}$ units (b) $23\sqrt{2}$ units
(c) $11\sqrt{2}$ units (d) None of these

22. If the points A (4, 3) and B (x, 5) are on the circle with centre O (2, 3), then the value of x is

(a) 0 (b) 1
(c) 2 (d) 3

23. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is **[NCERT Exemplar]**

(a) 5 units (b) 12 units
(c) 11 units (d) $(7+\sqrt{5})$ units

24. If three points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then λ equals

(a) 2 (b) −3
(c) −4 (d) None of these

25. If $AOBC$ is a rectangle whose three vertices are $A(0, 3)$, $O(0, 0)$ and $B(5, 0)$, then the length of its diagonal is **[NCERT Exemplar]**

(a) 5 units (b) 3 units
(c) $\sqrt{34}$ units (d) 4 units

26. The points (3, 2), (−2, − 3) and (2, 3) form a triangle name the type of triangle formed.

(a) equilateral (b) isosceles
(c) right angle (d) None of these

27. (5, − 2), (6, 4) and (7, − 2) are the vertices of an ……… triangle.

(a) equilateral (b) right angle
(c) isosceles (d) None of these

28. The points (− 4, 0), (4, 0) and (0, 3) are the vertices of a

(a) right angled triangle
(b) isosceles triangle
(c) equilateral triangle
(d) scalene triangle

29. The points (2, 3), (3, 4), (5, 6) and (4, 5) are the vertices of a

(a) Parallelogram (b) Triangle
(c) Square (d) None of these

30. The coordinates of the point which divides the line segment joining the points (4, − 3) and (9, 7) internally in the ratio 3 : 2 is

(a) (7, 3) (b) (3, 7)
(c) (35, 15) (d) (27, 21)

31. The point which divides the line segment joining the points (7, − 6) and (3, 4) in ratio 1 : 2 internally lies in the **[NCERT Exemplar]**

(a) I quadrant
(b) II quadrant
(c) III quadrant
(d) IV quadrant

32. If $P(9a-2,-b)$ divides line segment joining $A(3a+1,-3)$ and $B(8a,5)$ in the ratio $3:1$, then the values of a and b is *[NCERT Exemplar]*

(a) $a=-1, b=3$ (b) $a=-1, b=-3$
(c) $a=0, b=0$ (d) $a=1, b=-3$

33. The point $(-4, 6)$ divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$. The ratio is

(a) 1 : 2 (b) 7 : 2
(c) 2 : 7 (d) 4 : 1

34. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, then the value of a is

(a) -4 (b) -12
(c) 12 (d) -6

35. The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2, 3)$, $B(6, 7)$ and $C(8, 3)$ is

(a) (0, 1) (b) (0, −1)
(c) (−1, 0) (d) (1, 0)

36. If $x-2y+k=0$ is a median of the triangle whose vertices are at points $A(-1, 3)$, $B(0, 4)$ and $C(-5, 2)$, then the value of k is

(a) 2 (b) 4
(c) 6 (d) 8

37. The perpendicular bisector of the line segment joining the points $A(1, 5)$ and $B(4, 6)$ cuts the Y-axis at

(a) (0, 13) (b) (0, −13)
(c) (0, 12) (d) (13, 0)

38. The point .... lies on the perpendicular bisector of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$.

(a) (0, 0) (b) (0, 2)
(c) (2, 0) (d) (−2, 0)

39. A line intersects the Y-axis and X-axis at the points P and Q, respectively. If $(2, -5)$ is the mid-point of PQ, then the coordinates of P and Q are, respectively

(a) (0, −5) and (2, 0)
(b) (0, 10) and (− 4, 0)
(c) (0, 4) and (− 10, 0)
(d) (0, − 10) and (4, 0)

40. ΔABC with vertices $A(-2, 0)$, $B(2, 0)$ and $C(0, 2)$ is similar to ΔDEF with vertices $D(-4, 0)$, $E(4, 0)$ and $F(0, 4)$.

(a) True (b) False
(c) Can't say (d) Partially True/False

41. If (a, b) is the mid-point of the line segment joining the points $A(10, -6)$ and $B(k, 4)$ and $a-2b=18$, then the value of k is

(a) 30 (b) 22
(c) 4 (d) 40

42. Using section formula, check that the points $A(-3, -1)$, $B(1, 3)$ and $C(-1, 1)$ are collinear.

(a) Yes (b) No
(c) Can't say (d) None of these

43. The ratio, in which the Y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$ is

(a) 1 : 5 (b) 5 : 1
(c) 2 : 4 (d) None of these

44. Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2:3$.

(a) (1, 3) (b) (2, 6)
(c) (3, 4) (d) (4, 6)

45. The ratio in which the point $P(m, 6)$ divides the join $A(-4, 3)$ and $B(2, 8)$ is ....

(a) 2 : 3 (b) 1 : 2
(c) 3 : 2 (d) 2 : 1

46. If the points A (6, 1), B (8, 2), C (9, 4) and D (p, 3) are the vertices of a parallelogram, taken in order, then the value of p is

(a) 5 (b) 6
(c) 8 (d) 7

47. The coordinates of point A, where AB is the diameter of a circle whose centre is (3, − 4) and B is (1, 4) is

(a) (2, 0)
(b) (12, −5)
(c) (5, −12)
(d) None of the above

48. The coordinates of the point of trisection of the line segment joining (2, − 3) and (4, − 1). (when the point is near the point (2, − 3)) is

(a) (10/3, −5/3)
(b) (8/3, −7/3)
(c) (3, −2)
(d) None of the above

Matching Type

49. List-II gives the coordinates of the point P that divides the line segment joining the points in the given ratio given in List-I, match them correctly.

	List I		List II
P.	$A(-1, 3)$ and $B(-5, 6)$ internally in the ratio 1 : 2	1.	(7, 3)
Q.	$A(-2, 1)$ and $B(1, 4)$ internally in the ratio 2 : 1	2.	(0, 3)
R.	$A(1, 7)$ and $B(3, 4)$ internally in the ratio 3 : 2	3.	$\left(\frac{11}{5}, \frac{26}{5}\right)$
S.	$A(4, -3)$ and $B(8, 5)$ internally in the ratio 3 : 1	4.	$\left(-\frac{7}{3}, 4\right)$

Codes

	P	Q	R	S		P	Q	R	S
(a)	4	2	3	1	(b)	3	2	4	1
(c)	1	4	3	2	(d)	3	1	2	4

50. Match the following

	List I		List II
P.	Distance between (− 6, 7) and (− 1, − 5) is	1.	− 3, 7
Q.	The value of k for which the distance between $A(k, -5)$ and $B(2, 7)$ is 13 units	2.	$x + y = 5$
R.	(x, y) is equidistant from (5, 1) and (−1, 5), if	3.	$3x = 2y$
S.	(x, y), (2, 3) and (4, 1) are collinear, if	4.	13 units

Codes

	P	Q	R	S		P	Q	R	S
(a)	3	4	2	1	(b)	1	2	4	3
(c)	4	1	3	2	(d)	3	2	4	1

Assertion-Reasoning MCQs

Directions (Q. Nos. 46-54) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true; R is False.
(d) A is false; R is true.

51. **Assertion** There is no such point or X-axis which are at a distance c ($c < 3$) from the point (2, 3)

Reason The distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

52. **Assertion** The distance of a points $P(x, y)$ from the origin is $\sqrt{x^2 - y^2}$.

Reason If $P(-1, 1)$ is the mid-point of the line segment joining $A(-3, b)$ and $B(1, b + 4)$, then value of b is −1.

53. **Assertion** If the points A (4, 3) and B (x, 5) are on the circle with centre O (2 , 3), then find the value of x is 2.

Reason If three points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then λ equals to $\pm\sqrt{2}$.

54. **Assertion** Three points A, B, C are such that $AB + BC > AC$, then they are collinear.

Reason Three points are collinear if they lie on a straight line.

55. **Assertion** Points $A(6, 4)$, $B(-4, -6)$ and $C(4, 6)$ are such that $AB = \sqrt{200}$, $BC = \sqrt{208}$, $AC = \sqrt{8}$.

Since, $AB + BC > AC$, points A, B and C form a triangle.

Reason If $BC^2 = AB^2 + AC^2$, then ΔABC is a right triangle, right angled at A.

56. **Assertion** Points (3, 2), (−2, − 3) and (2, 3) form a right triangle.

Reason If (x, y) is equidistant from (3, 6) and (−3, 4), then $3x + y = 5$.

57. **Assertion** In quadrilateral $ABCD$, if $AB = BC = CD = DA$ and $AC = BD$, then $ABCD$ is a square.

Reason A quadrilateral is a square if all its sides are equal and the diagonals are equal.

58. **Assertion** The distance between the points $(10\cos 30°, 0)$ and $(0, 10\cos 60°)$ is 10 units

Reason Mid-point of line segment joining (a, b) and (c, d) is given by $\left(\frac{a-c}{2}, \frac{b-d}{2}\right)$.

59. **Assertion** The coordinates of the points which divide the line segment joining $A(2, -8)$ and $B(-3, -7)$ into three equal parts are $\left(\frac{1}{3}, \frac{-23}{3}\right)$ and $\left(\frac{-4}{3}, \frac{-22}{3}\right)$.

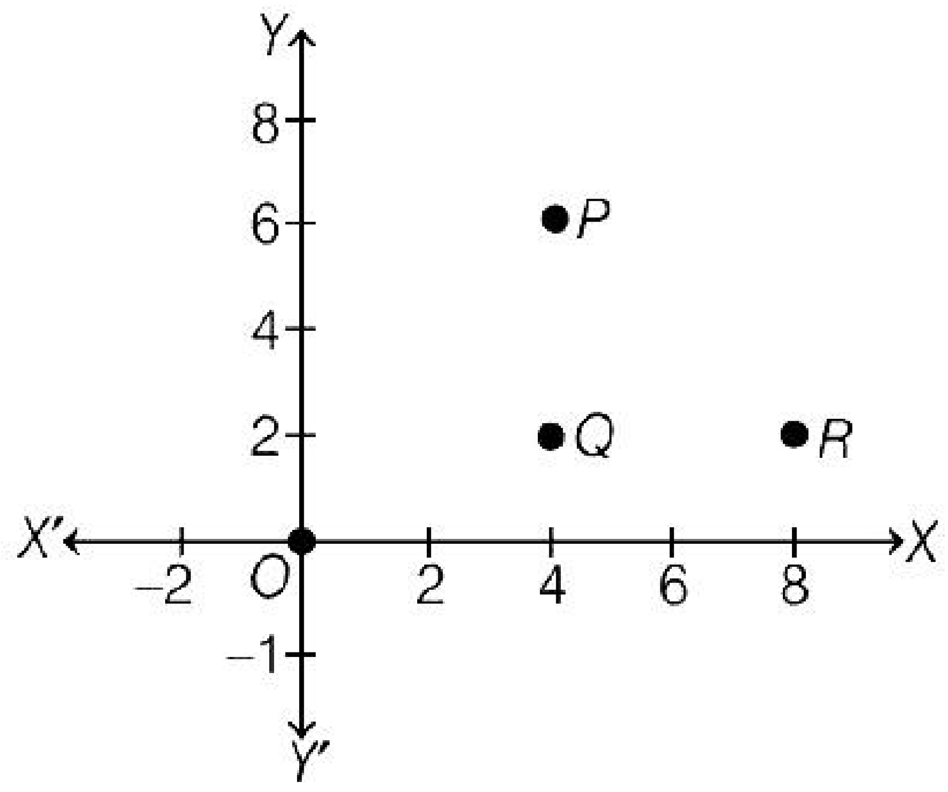
Reason The points which divide AB in the ratio 1 : 3 and 3 : 1 are called points to trisection of AB.

60. **Assertion** Mid-point of a line segment divides line in the ratio 1 : 1.

Reason If area of triangle is zero that means points are collinear.

Case Based MCQs

61. Malika and Karishma are friends living on the same street in Govindpuri. Karishma's house is at the intersection of one street with another street on which there is a temple. They both study in the same school and that is not far from Karishma's house.



Suppose the school is situated at the point O, i.e., the origin, Malika's house is at P Karishma's house is at Q and temple is at R.

Based on the above information, answer the following questions.

(i) How far is Malika's house from Karishma's house?

(a) 3 units (b) 4 units
(c) 5 units (d) 2 units

(ii) How far is the temple from Karishma's house?

(a) 3 units (b) 2 units
(c) 5 units (d) 4 units

(iii) How far is the temple from Malika's house?

(a) $2\sqrt{2}$ units (b) $3\sqrt{2}$ units
(c) $4\sqrt{2}$ units (d) None of these

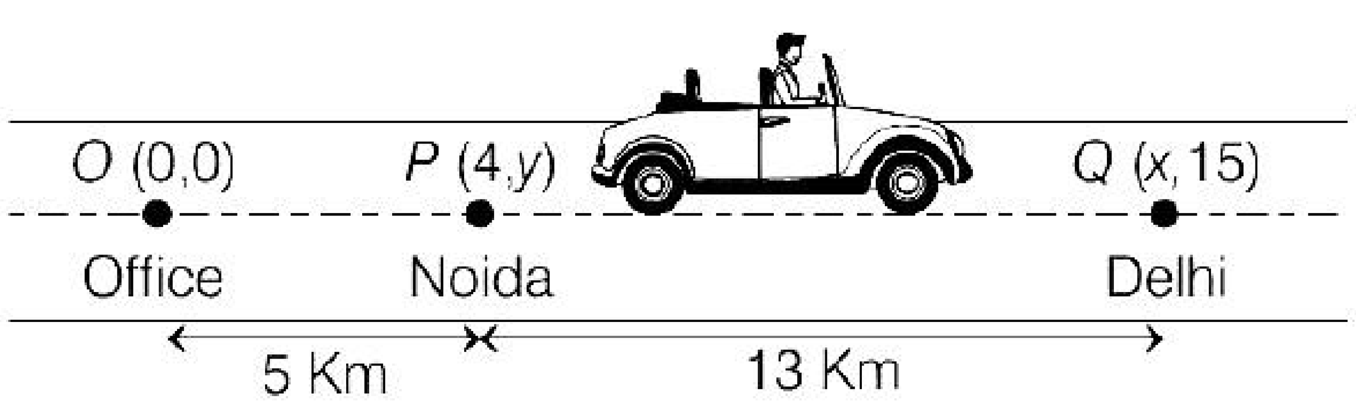
(iv) Which of the following is true?

(a) *PQR* forms a scalene triangle
(b) *PQR* forms an isosceles triangle
(c) *PQR* forms an equilateral triangle
(d) None of the above

(v) How far is the school from Malika's house?

(a) $2\sqrt{13}$ units
(b) $\sqrt{5}$ units
(c) $(\sqrt{13}+\sqrt{5})$ units
(d) $(\sqrt{13}-\sqrt{5})$ units

62. Anmol is driving his car on a straight road towards East from his office to Noida and then to Delhi. At some point in between Noida and Delhi, he suddenly realises that there is not enough Petrol for the journey. Also, there is no Petrol pump on the road between these two cities.



Based on the above information, answer the following questions.

(i) The value of y is equal to

(a) 2 (b) 3
(c) 4 (d) 5

(ii) The value of x is equal to

(a) 4 (b) 5
(c) 8 (d) 9

(iii) If M is any point exactly in between Noida and Delhi, then coordinates of M are

(a) (3, 3) (b) (6.5, 9)
(c) (5, 5) (d) (6, 6)

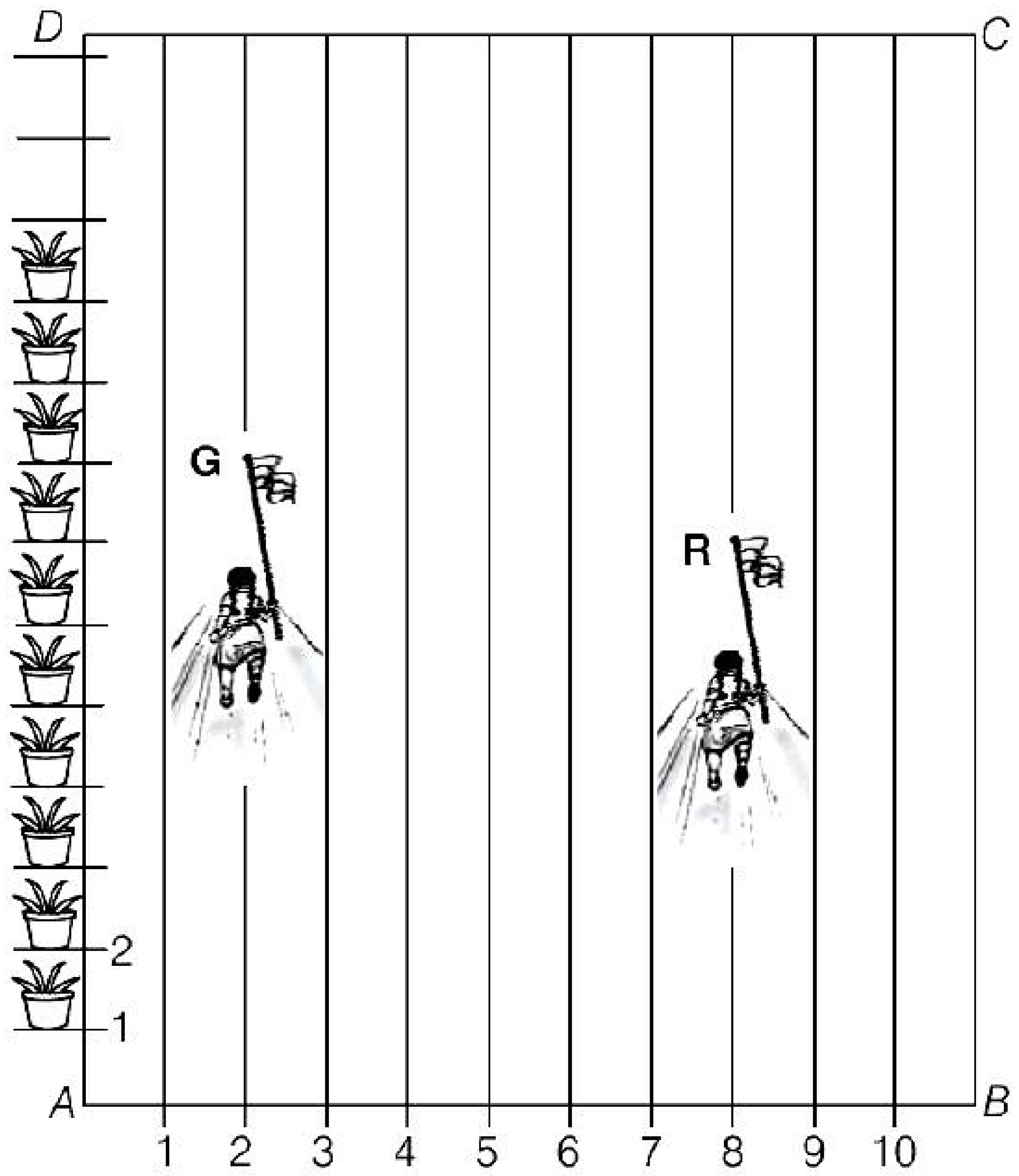
(iv) The ratio in which Noida divides the line segment joining the office and Delhi is

(a) 1 : 4 (b) 2 : 1
(c) 3 : 2 (d) 2 : 3

(v) If Anmol analyse the CNG at the point M (the mid point of Noida-Delhi), then what should be his decision?

(a) Should he travel back to office
(b) Should try his luck to move towards Delhi
(c) Should be travel back to Noida
(d) None of the above

63. In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground *ABCD*, 100 flowerpots have been placed at a distance of 1 m from each other along *AD*, as shown in given figure below. Niharika runs 1/4 th the distance *AD* on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th distance *AD* on the eighth line and posts a red flag. ***[CBSE Question Bank]***



(i) Find the position of green flag

(a) (2, 25) (b) (2, 0.25)
(c) (25, 2) (d) (0, – 25)

(ii) Find the position of red flag

(a) (8, 0) (b) (20, 8)
(c) (8, 20) (d) (8, 0.2)

(iii) What is the distance between both the flags?

(a) $\sqrt{41}$ m (b) $\sqrt{11}$ m
(c) $\sqrt{61}$ m (d) $\sqrt{51}$ m

(iv) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

(a) (5, 22.5) (b) (10, 22)
(c) (2, 8.5) (d) (2.5, 20)

(v) If Joy has to post a flag at 1/4th distance from green flag ,in the line segment joining the green and red flags, then where should he post his flag?

(a) (3.5, 24)
(b) (0.5, 12.5)
(c) (2.25, 8.5)
(d) (25, 20)

64. In a cinema hall, peoples are seated at a distance of 1 m from each other, to maintain the social distance due to CORONA virus pandemic. Let three peoples sit at points P, Q and R whose coordinates are (6, – 2), (9, 4) and (10, 6) respectively.

Based on the above information, answer the following questions.

(i) The distance between P and R is

(a) $\sqrt{5}$ units (b) $4\sqrt{5}$ units
(c) $3\sqrt{5}$ units (d) None of these

(ii) If *a* TC at the point I, lying on the straight line joining Q and R such that it divides the distance between them in the ratio of 1 : 2. Then coordinates of I are

(a) $\left(\frac{22}{3}, \frac{11}{3}\right)$ (b) $\left(\frac{28}{3}, \frac{14}{3}\right)$
(c) (6, 1) (d) (9, 1)

(iii) The mid-point of the line segment joining P and R is

(a) (1, 6) (b) (6, 1)
(c) $\left(\frac{11}{2}, 0\right)$ (d) None of these

(iv) The ratio in which Q divides the line segment joining P and R is

(a) 2 : 1 (b) 3 : 1
(c) 1 : 2 (d) None of these

(v) The points P, Q and R lies on

(a) a straight line
(b) an equilateral triangle
(c) a scalene triangle
(d) an isosceles triangle

65. To raise social awareness about hazards of cancer, an organisation decided to start a campaign. 5 students were asked to prepare campaign banners in the shape of a triangle. The vertices of the triangle are, $A(0, 6)$, $B(6, 6)$ and $C(1, 1)$.

Based on the above information, answer the following questions.

(i) The coordinates of centroid of ΔABC are

(a) $\left(\frac{2}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{7}{3}, \frac{13}{3}\right)$

(c) $\left(\frac{-2}{3}, \frac{7}{3}\right)$ (d) $\left(\frac{7}{3}, \frac{2}{3}\right)$

(ii) If S be the mid-point of line joining A and B, then coordinates of S are

(a) (4, 0) (b) (2, 0)

(c) (3, 6) (d) (0, 4)

(iii) If T be the mid-point of the line joining C and B, then coordinates of T are

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (d) None of these

(iv) If U be the mid-point of line joining C and A then coordinates of U are

(a) $\left(-\frac{5}{2}, \frac{3}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{7}{2}\right)$

(c) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (d) $\left(\frac{5}{2}, \frac{3}{2}\right)$

(v) The coordinates of centroid of ΔSTU are

(a) $\left(\frac{2}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{3}\right)$

(c) $\left(-\frac{2}{3}, \frac{7}{3}\right)$ (d) $\left(\frac{7}{3}, \frac{13}{3}\right)$

ANSWERS

Multiple Choice Questions

1. (a)	2. (b)	3. (a)	4. (d)	5. (a)	6. (b)	7. (b)	8. (c)	9. (d)	10. (b)
11. (c)	12. (a)	13. (a)	14. (a)	15. (d)	16. (b)	17. (d)	18. (c)	19. (a)	20. (a)
21. (c)	22. (c)	23. (b)	24. (d)	25. (c)	26. (c)	27. (c)	28. (b)	29. (a)	30. (a)
31. (d)	32. (d)	33. (c)	34. (b)	35. (b)	36. (d)	37. (a)	38. (a)	39. (d)	40. (a)
41. (b)	42. (a)	43. (b)	44. (a)	45. (c)	46. (d)	47. (c)	48. (b)	49. (a)	50. (c)

Assertion-Reasoning MCQs

51. (a)	52. (d)	53. (c)	54. (d)	55. (b)	56. (b)	57. (a)	58. (c)	59. (c)	60. (b)

Case Based MCQs

61. (i) - (b); (ii) - (d); (iii) - (c); (iv) - (b); (v) - (a)

62. (i) - (b); (ii) - (d); (iii) - (b); (iv) - (a); (v) - (b)

63. (i) - (a); (ii) - (c); (iii) - (c); (iv) - (a); (v) - (a)

64. (i) - (b); (ii) - (b); (iii) - (d); (iv) - (b); (v) - (a)

65. (i) - (b); (ii) - (c); (iii) - (d); (iv) - (b); (v) - (d)

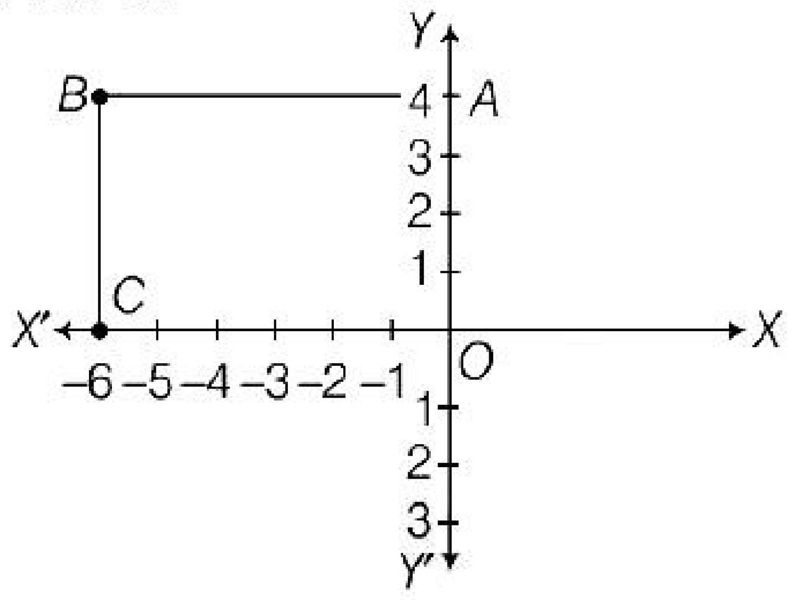
SOLUTIONS

1. Here, the point A lies on side OA, which is at a distance of 4 units from the intersection point O of the lines OX and OY, so its position will be (4, 0) and point C lies on the side OC at a distance of 4 units from the intersection point O, so its position will be (0, 4). Point B lies 4 units away from both the sides (or axes), so its position will be (4, 4).

2. The y-coordinate is also called the ordinate so,
   for P : ordinate = 3
   for Q : ordinate = 4
   for R : ordinate = − 3
   for S : ordinate = − 1

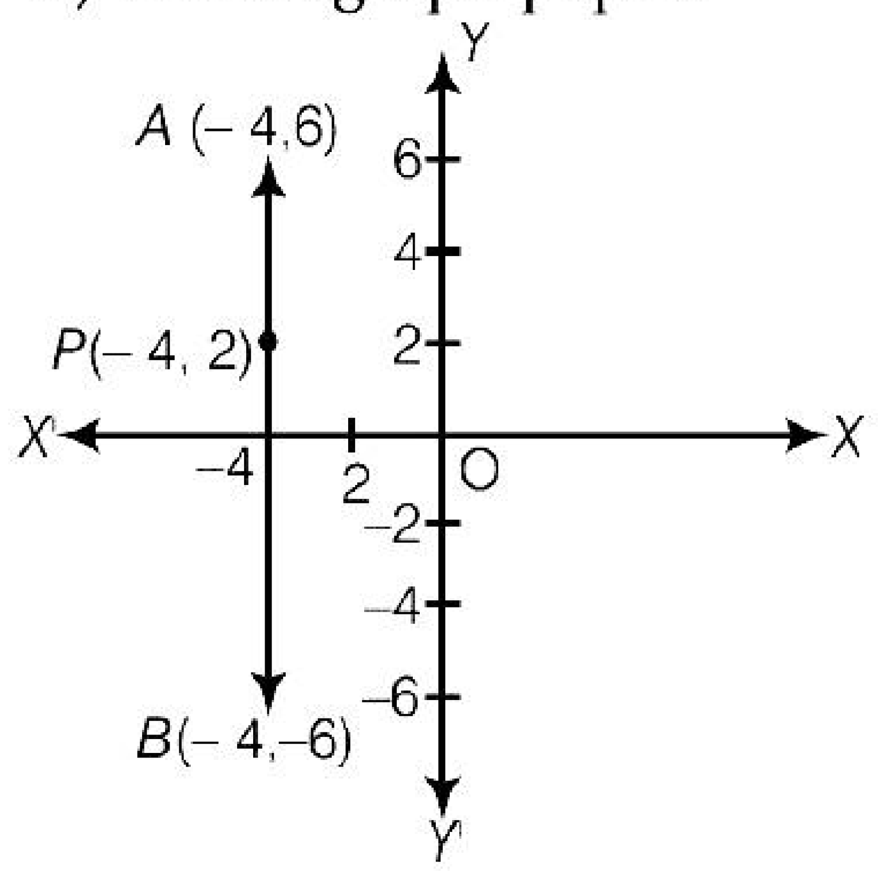
3. It is clear from the graph that point P has 3 unit distance from X-axis in negative y-direction and 2 unit distance from Y-axis in positive x-direction.
   Hence, coordinates of P are (2, − 3).

4. According to the question, the figure would be given as



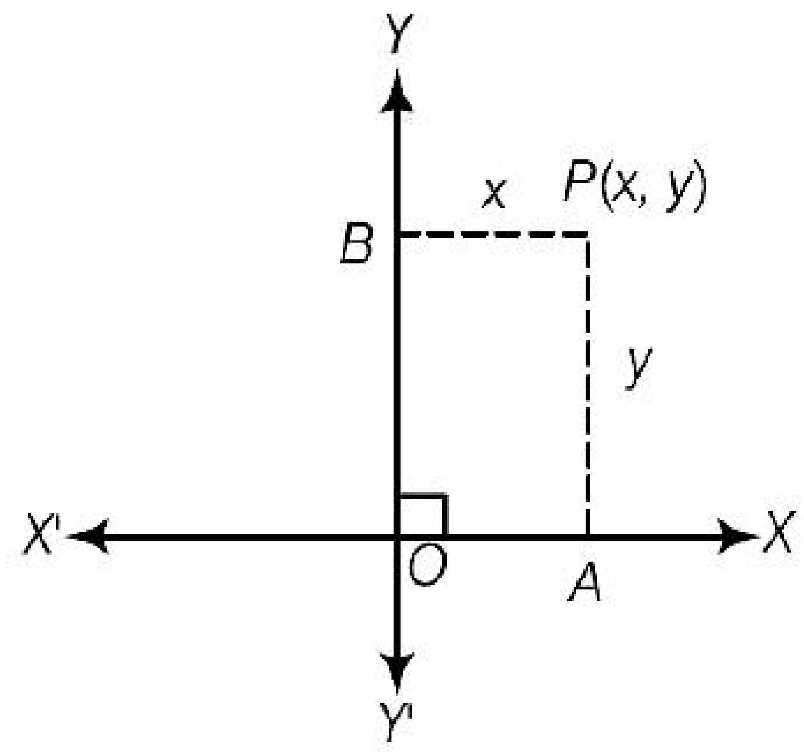
   Coordinates of point O = (0, 0)
   Coordinates of point A = (0, 4)
   Coordinates of point B = (−6, 4)
   Coordinates of point C = (−6, 0)

5. We plot all the points $P(-4, 2)$, $A(-4, 6)$ and $B(-4, -6)$ on the graph paper.



   From the figure, point $P(-4, 2)$ lies on the line segment joining the points $A(-4, 6)$ and $B(-4, -6)$.

6. We know that, if (x, y) is any point on the cartesian plane in first quadrant.
   Then, x = Perpendicular distance from Y-axis
   and y = Perpendicular distance from X-axis



   Distance of the point $P(2, 3)$ from the X-axis = Ordinate of a point $P(2, 3)$
   = 3 units.

7. Here, $x_1 = -6$, $y_1 = 7$, $x_2 = -1$ and $y_2 = -5$
   ∴ Distance between two points,
   $$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
   [by distance formula]
   $$= \sqrt{(-1+6)^2 + (-5-7)^2}$$
   $$= \sqrt{(5)^2 + (-12)^2}$$
   $$= \sqrt{25 + 144} = \sqrt{169} = 13 \text{ units}$$

8. The given points are $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$.
   Here, $x_1 = a\cos\theta + b\sin\theta$, $y_1 = 0$
   $x_2 = 0$, $y_2 = a\sin\theta - b\cos\theta$
   ∴ Required distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
   $$= \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (0 - a\sin\theta + b\cos\theta)^2}$$
   $$= \sqrt{(a\cos\theta + b\sin\theta)^2 + (b\cos\theta - a\sin\theta)^2}$$

$$= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + b^2\cos^2\theta + a^2\sin^2\theta - 2ab\sin\theta\cos\theta}$$

$$= \sqrt{\cos^2\theta(a^2+b^2) + \sin^2\theta(a^2+b^2)}$$

$$= \sqrt{(a^2+b^2)(\cos^2\theta+\sin^2\theta)}$$

$$= \sqrt{a^2+b^2} \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

9. Let points are $A(2, y)$ and $B(-4, 3)$.
Here, $(x_1, y_1) = (2, y)$ and $(x_2, y_2) = (-4, 3)$
$\therefore$ Distance between two points,

$$AB = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

[by distance formula]

$$\Rightarrow AB = \sqrt{(2+4)^2 + (y-3)^2}$$

$$\Rightarrow 10 = \sqrt{(6)^2 + y^2 + 9 - 6y}$$

$[\because AB = 10$ and $(a-b)^2 = a^2 + b^2 - 2ab]$

On squaring both sides, we get

$$\Rightarrow (10)^2 = (6)^2 + y^2 + 9 - 6y$$

$$\Rightarrow 100 = 36 + y^2 + 9 - 6y$$

$$\Rightarrow 100 = 45 + y^2 - 6y$$

$$\Rightarrow y^2 - 6y - 55 = 0$$

$$\Rightarrow y^2 - 11y + 5y - 55 = 0$$

[by factorisation]

$$\Rightarrow y(y-11) + 5(y-11) = 0$$

$$\Rightarrow (y-11)(y+5) = 0$$

$$\Rightarrow y - 11 = 0 \text{ or } y + 5 = 0$$

$$\Rightarrow y = 11 \text{ or } y = -5$$

Hence, the required values of y are 11 and –5.

10. We know that, the points lies on perpendicular bisector of the line segment joining the two points is equidistant from these two points.

$$\therefore PA = \sqrt{(-1-0)^2 + (1-2)^2}$$

$$= \sqrt{1+1} = \sqrt{2} \text{ units}$$

$$PB = \sqrt{(3-0)^2 + (3-2)^2} = \sqrt{9+1} = 10 \text{ units}$$

$\because \quad PA \neq PB$

So, the point P does not lie on the perpendicular bisctor of AB.

11. According to the question, the distance between the points $(4, p)$ and $(1, 0) = 5$

i.e. $\sqrt{(1-4)^2 + (0-p)^2} = 5$

$\left[\because \text{distance between the points } (x_1, y_1) \text{ and } (x_2, y_2),\ d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\right]$

$$\Rightarrow \sqrt{(-3)^2 + p^2} = 5$$

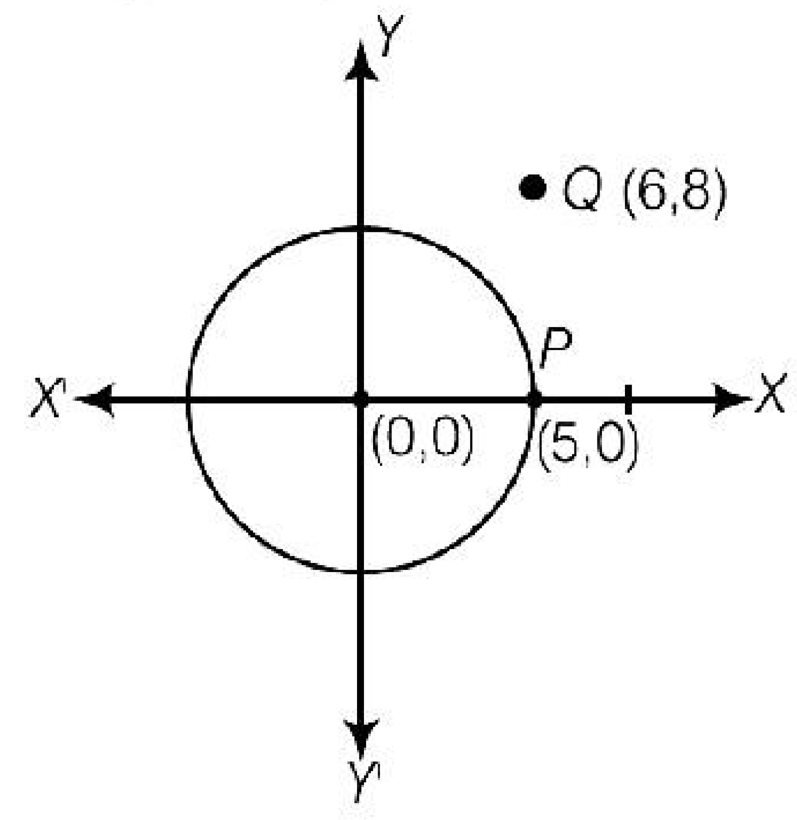
$$\Rightarrow \sqrt{9 + p^2} = 5$$

On squaring both the sides, we get

$$9 + p^2 = 25 \Rightarrow p^2 = 16 \Rightarrow p = \pm 4$$

Hence, the required value of p is ± 4.

12. First, we draw a circle and a point from the given information.



Now, distance between origin i.e. $O(0, 0)$ and $P(5, 0)$, $OP = \sqrt{(5-0)^2 + (0-0)^2}$

$\left[\because \text{Distance between two points } (x_1, y_1) \text{ and } (x_2, y_2),\ d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\right]$

$$= \sqrt{5^2 + 0^2} = 5 \text{ units}$$

= Radius of circle and distance between origin $O(0, 0)$

and $Q(6, 8)$, $OQ = \sqrt{(6-0)^2 + (8-0)^2}$

$$= \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$$

$$= \sqrt{100} = 10 \text{ units}$$

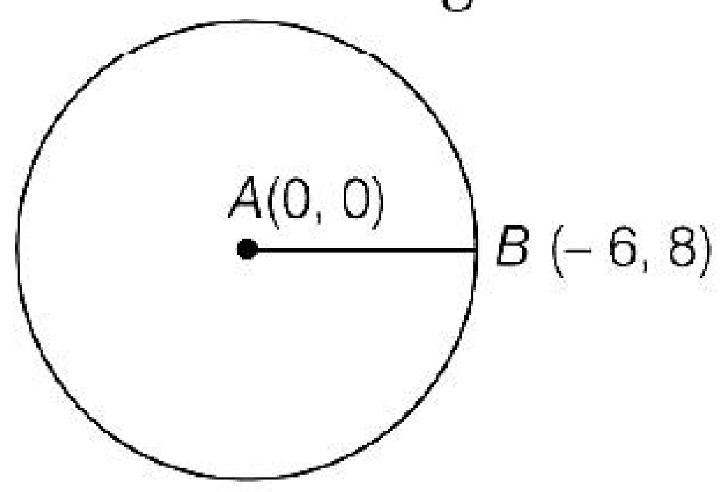
We know that, if the distance of any point from the centre is less than/equal to/ more than the radius, then the point is inside/on/outside the circle, respectively.

Here, we see that, $OQ > OP$

Hence, it is true that point Q (6, 8), lies outside the circle.

13. Let $A(0, 0)$ and $B(-6, 8)$ be the given points.

Now, radius of the circle is same as the distance of the line segment AB.



$\therefore \quad AB = \sqrt{(-6-0)^2 + (8-0)^2}$

$= \sqrt{36 + 64} = \sqrt{100} = 10$

Hence, radius of the circle is 10 units.

14. Let $A = (1, -1)$, $B = (5, 2)$ and $C = (9, 5)$.

Then, $AB = \sqrt{(5-1)^2 + (2+1)^2}$

$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$= \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9}$

$= \sqrt{25} = 5$ units

$BC = \sqrt{(9-5)^2 + (5-2)^2}$

$= \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$ units

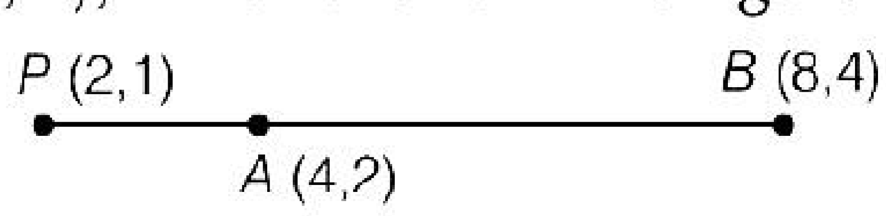
and $AC = \sqrt{(9-1)^2 + (5+1)^2}$

$= \sqrt{(8)^2 + (6)^2} = \sqrt{100} = 10$ units

$\because$ Here, $AC = AB + BC$

$\therefore$ A, B and C are collinear points.

15. Given that the point $P(2, 1)$ lies on the line segment joining the points $A(4, 2)$ and $B(8, 4)$, which shows in the figure below



Now, distance between $A(4, 2)$ and $P(2, 1)$,

$AP = \sqrt{(2-4)^2 + (1-2)^2}$

$\left[\because \text{ distance between two points two points } (x_1, y_1) \text{ and } B(x_2, y_2), \ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$= \sqrt{(-2)^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5}$ units

Distance between $A(4, 2)$ and $B(8, 4)$,

$AB = \sqrt{(8-4)^2 + (4-2)^2} = \sqrt{(4)^2 + (2)^2}$

$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$ units

Distance between $B(8, 4)$ and $P(2, 1)$,

$BP = \sqrt{(8-2)^2 + (4-1)^2}$

$= \sqrt{6^2 + 3^2} = \sqrt{36 + 9}$

$= \sqrt{45} = 3\sqrt{5}$ units

$\therefore \quad AB = 2\sqrt{5} = 2AP$

$\Rightarrow \quad AP = \dfrac{AB}{2}$

Hence, required condition is $AP = \dfrac{AB}{2}$.

16. Given, point $P(x, y)$ is equidistant from the points $A(5, 1)$ and $B(1, 5)$.

So, $\quad AP = BP$

$\Rightarrow \quad AP^2 = BP^2$ [on squaring both sides]

$\Rightarrow \quad (x-5)^2 + (y-1)^2 = (x-1)^2 + (y-5)^2$

$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y$

$= x^2 + 1 - 2x + y^2 + 25 - 10y$

$[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow \quad -10x + 2x = -10y + 2y$

$\Rightarrow \quad -8x = -8y$

$\Rightarrow \quad x = y$

17. Let $A(x, 0)$ be any point on X-axis, which is equidistant from points $B(1, 3)$ and $(-1, 2)$.

$AB = AC$

$\Rightarrow \quad AB^2 - AC^2$

[squaring both sides]

$\Rightarrow (1-x)^2 + (3-0)^2 = (-1-x)^2 + (2-0)^2$

$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$\Rightarrow \quad 1 + x^2 - 2x + 9 = 1 + x^2 + 2x + 4$

$\Rightarrow \quad 9 - 4 = 2x + 2x$

$\Rightarrow \quad 5 = 4x \Rightarrow x = 5/4$

Hence, point on X-axis is $(5/4, 0)$.

18. Let $A(x, 0)$ be any point on the X-axis, which is equidistant from points $B(7, 6)$ and $C(-3, 4)$.

$\therefore \quad AB = AC$

$\Rightarrow \quad AB^2 = AC^2$ [on squaring both sides]

$\Rightarrow (7-x)^2 + (6-0)^2 = (-3-x)^2 + (4-0)^2$

$[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}]$

$\Rightarrow 49 + x^2 - 14x + 36 = 9 + x^2 + 6x + 16$

$[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow \quad 20x = 60 \Rightarrow x = 3$

Hence, the required point is $A(3, 0)$.

19. Let the coordinate of the point which is equidistant from the three vertices $O(0, 0)$, $A(0, 2y)$ and $B(2x, 0)$ is $P(h, k)$.

Then, $\quad PO = PA = PB$

$\Rightarrow \quad PO^2 = PA^2 = PB^2$ ...(i)

By distance formula,

$[\sqrt{(h-0)^2 + (k-0)^2}]^2$

$= [\sqrt{(h-0)^2 + (k-2y)^2}]^2$

$= [\sqrt{(h-2x)^2 + (k-0)^2}]^2$

$\Rightarrow \quad h^2 + k^2 = h^2 + (k-2y)^2$

$= (h-2x)^2 + k^2$ ...(ii)

Taking first two equations, we get

$h^2 + k^2 = h^2 + (k-2y)^2$

$\Rightarrow \quad k^2 = k^2 + 4y^2 - 4yk \Rightarrow 4y(y-k) = 0$

$\Rightarrow \quad y = k$ $[\because y \neq 0]$

Taking first and third equations, we get

$h^2 + k^2 = (h-2x)^2 + k^2$

$\Rightarrow \quad h^2 = h^2 + 4x^2 - 4xh$

$\Rightarrow \quad 4x(x-h) = 0$

$\Rightarrow \quad x = h$ $[\because x \neq 0]$

$\therefore$ Required points $= (h, k) = (x, y)$

20. Let point on Y-axis be $P(0, k)$. Then,

$PA^2 = PB^2$

$\Rightarrow (6-0)^2 + (5-k)^2 = (-4-0)^2 + (3-k)^2$

$\Rightarrow 36 + 25 + k^2 - 10k = 16 + 9 + k^2 - 6k$

$\Rightarrow \quad 4k = 36 \Rightarrow k = 9$

Hence, coordinates of point on Y-axis are (0, 9).

21. The distance between ends points of the diameter gives the value of the diameter.

Here, the points are (24, 1) and (2, 23)

$\therefore \quad d = \sqrt{(2-24)^2 + (23-1)^2}$

$= \sqrt{(-22)^2 + (22)^2}$

$= \sqrt{(22)^2(1+1)}$

$= 22\sqrt{2}$ units

$\therefore$ Radius of a circle $r = \frac{d}{2} = \frac{22\sqrt{2}}{2}$

$= 11\sqrt{2}$ units

22. Since, A and B lie on the circle having centre O.

$OA = OB$

$\Rightarrow \sqrt{(4-2)^2 + (3-3)^2} = \sqrt{(x-2)^2 + (5-3)^2}$

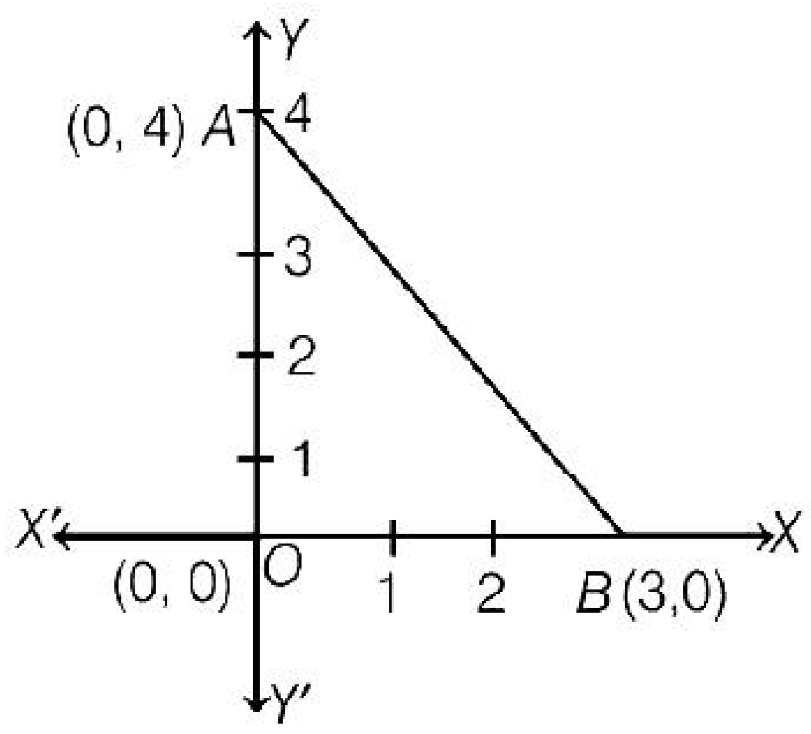
$\Rightarrow \quad 2 = \sqrt{(x-2)^2 + 4}$

$\Rightarrow \quad 4 = (x-2)^2 + 4$

$\Rightarrow \quad (x-2)^2 = 0$

$\Rightarrow \quad x = 2$

23. We plot the vertices of a triangle i.e., (0, 4), (0, 0) and (3, 0) on the paper shown as given below



Now, perimeter of ΔAOB = Sum of the length of all its sides

$= d(AO) + d(OB) + d(AB)$

$\because$ Distance between the points (x_1, y_1) and (x_2, y_2), is

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= Distance between $A(0, 4)$ and $O(0, 0)$ + Distance between $O(0, 0)$ and $B(3, 0)$ + Distance between $A(0, 4)$ and $B(3, 0)$

$= \sqrt{(0-0)^2 + (0-4)^2} + \sqrt{(3-0)^2 + (0-0)^2} + \sqrt{(3-0)^2 + (0-4)^2}$

$= \sqrt{0+16} + \sqrt{9+0} + \sqrt{(3)^2 + (4)^2}$

$= 4 + 3 + \sqrt{9+16}$

$= 7 + \sqrt{25} = 7 + 5 = 12$

Hence, the required perimeter of triangle is 12 units.

24. Let the given points are $A(0, 0)$, $B(3, \sqrt{3})$ and $C(3, \lambda)$.

Since, ΔABC is an equilateral triangle, therefore

$$AB = AC$$

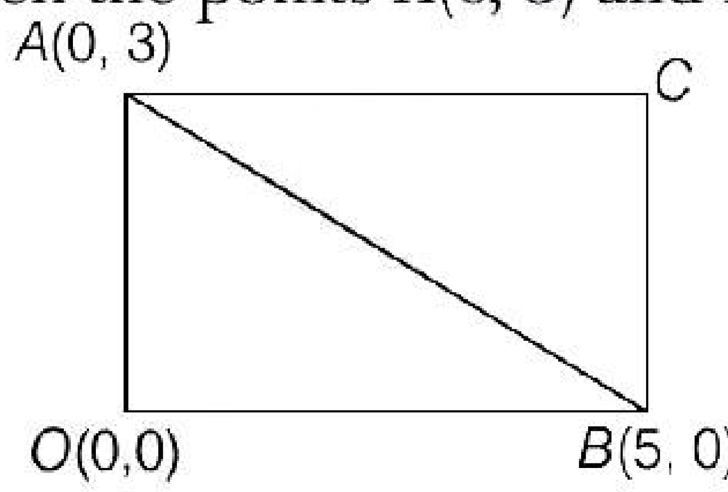
$$\Rightarrow \sqrt{(3-0)^2 + (\sqrt{3}-0)^2} = \sqrt{(3-0)^2 + (\lambda - 0)^2}$$

$$\Rightarrow 9 + 3 = 9 + \lambda^2$$

$$\Rightarrow \lambda^2 = 3$$

$$\Rightarrow \lambda = \pm\sqrt{3}$$

25. Now, length of the diagonal AB = Distance between the points $A(0, 3)$ and $B(5, 0)$.



$\because$ Distance between the points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0$, $y_1 = 3$ and $x_2 = 5$, $y_2 = 0$

$\therefore$ Distance between the points $A(0, 3)$ and $B(5, 0)$

$$AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{25+9} = \sqrt{34}$$

Hence, the required length of its diagonal is $\sqrt{34}$ units.

26. Let the points are $A(3,2)$, $B(-2,-3)$ and $C(2,3)$.

Then, $AB = \sqrt{(-2-3)^2 + (-3-2)^2}$

$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$= \sqrt{(-5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50}$

= 7.07 units (approx)

$BC = \sqrt{(2+2)^2 + (3+3)^2} = \sqrt{(4)^2 + (6)^2}$

$= \sqrt{16+36} = \sqrt{52} = 7.21$ units (approx)

and $CA = \sqrt{(3-2)^2 + (2-3)^2}$

$= \sqrt{(1)^2 + (-1)^2} = \sqrt{1+1}$

$= \sqrt{2} = 1.41$ (approx)

Also, $(\sqrt{52})^2 = (\sqrt{50})^2 + (\sqrt{2})^2$

$\Rightarrow BC^2 = AB^2 + CA^2$

So, by converse of Pythagoras theorem, $\angle A = 90°$

Hence, ΔBAC is a right angled triangle.

27. Let $P(5, -2)$, $Q(6, 4)$ and $R(7, -2)$ are the given points.

Then, $PQ = \sqrt{(6-5)^2 + (4+2)^2}$

$\left[\because d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$= \sqrt{1+36} = \sqrt{37}$ units

$QR = \sqrt{(7-6)^2 + (-2-4)^2}$

$= \sqrt{1+36} = \sqrt{37}$ units

Since, $PQ = QR$

$\therefore$ ΔPQR is an isosceles triangle.

28. Let $A(-4, 0)$, $B(4, 0)$, $C(0, 3)$ are the given vertices.

Now, distance between $A(-4, 0)$ and $B(4, 0)$,

$AB = \sqrt{[4-(-4)]^2 + (0-0)^2}$

$\left[\because \text{distance between two points } (x_1, y_1) \text{ and } (x_2, y_2),\ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$= \sqrt{(4+4)^2} = \sqrt{8^2} = 8$ units

Distance between $B(4, 0)$ and $C(0, 3)$,

$$BC = \sqrt{(0-4)^2 + (3-0)^2}$$
$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

Distance between $A(-4, 0)$ and $C(0, 3)$,

$$AC = \sqrt{[0-(-4)]^2 + (3-0)^2}$$
$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

$\because \quad BC = AC$

Hence, ΔABC is an isosceles triangle because an isosceles triangle has two sides equal.

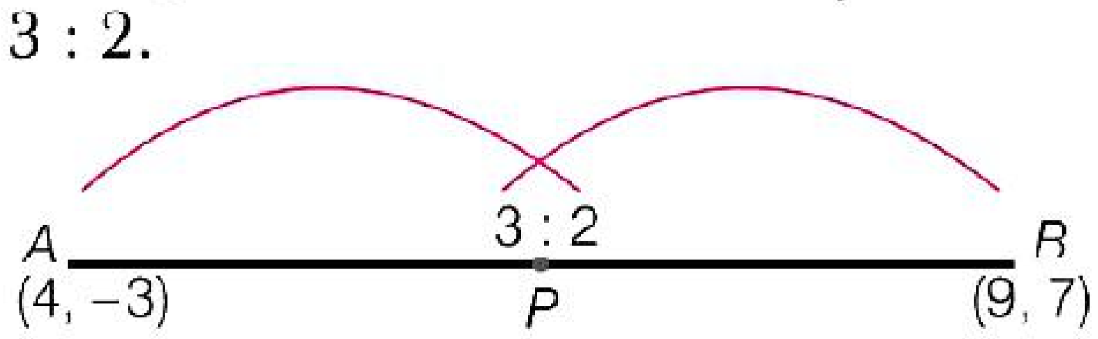
29. Let the points are $A(2, 3)$, $B(3, 4)$, $C(5, 6)$ and $D(4, 5)$.
Then, by distance formula

$$AB = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{(1)^2 + (1)^2}$$
$$= \sqrt{2} \text{ units}$$
$$BC = \sqrt{(5-3)^2 + (6-4)^2}$$
$$= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4}$$
$$= \sqrt{8} = 2\sqrt{2} \text{ units}$$
$$CD = \sqrt{(4-5)^2 + (5-6)^2}$$
$$= \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \text{ units}$$
$$\text{and } AD = \sqrt{(4-2)^2 + (5-3)^2} = \sqrt{(2)^2 + (2)^2}$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$

Here, $AB = CD$ and $AD = BC$ i.e. the opposite sides are equal. So, given points are vertices of a parallelogram.

30. Let $P(x, y)$ be the required point.

Then, P divides AB internally in the ratio $3 : 2$.



Here, $\frac{m_1}{m_2} = \frac{3}{2}$ and $(x_1, y_1) = (4, -3)$,

$(x_2, y_2) = (9, 7)$

Then, $P(x, y) =$

$$P\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

[by section formula]

$$= P\left(\frac{3 \times 9 + 2 \times 4}{3+2}, \frac{3 \times 7 + 2 \times (-3)}{3+2}\right)$$
$$= P\left(\frac{27+8}{5}, \frac{21-6}{5}\right) = P\left(\frac{35}{5}, \frac{15}{5}\right)$$
$$= P(7, 3)$$

Therefore, $(7, 3)$ is the required point.

31. If $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio

$m : n$, then $x = \frac{mx_2 + nx_1}{m+n}$ and $y = \frac{my_2 + ny_1}{m+n}$

Given that, $x_1 = 7$, $y_1 = -6$, $x_2 = 3$, $y_2 = 4$, $m = 1$ and $n = 2$

$$\therefore \quad x = \frac{1(3) + 2(7)}{1+2}, \; y = \frac{1(4) + 2(-6)}{1+2}$$

[by section formula]

$$\Rightarrow \quad x = \frac{3+14}{3}, \; y = \frac{4-12}{3}$$
$$\Rightarrow \quad x = \frac{17}{3}, \; y = -\frac{8}{3}$$

So, $(x, y) = \left(\frac{17}{3}, -\frac{8}{3}\right)$ lies in IV quadrant.

[since, in IV quadrant, x-coordinate is positive and y-coordinate is negative]

32. Let $P(9a - 2, -b)$ divides AB internally in the ratio $3 : 1$.

By section formula,

$$9a - 2 = \frac{3(8a) + 1(3a+1)}{3+1}$$

$\left[\because\right.$ internal section formula, the coordinates of point P divides the line segment joining the point (x_1, y_1) and (x_2, y_2) in the ratio $m_1 : m_2$ internally is $\left.\left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2}\right)\right]$

$$\text{and} \quad -b = \frac{3(5) + 1(-3)}{3+1}$$
$$\Rightarrow \quad 9a - 2 = \frac{24a + 3a + 1}{4}$$
$$\text{and} \quad -b = \frac{15-3}{4}$$

$$\Rightarrow \quad 9a - 2 = \frac{27a + 1}{4}$$

and $\quad -b = \frac{12}{4}$

$$\Rightarrow \quad 36a - 8 = 27a + 1 \text{ and } b = -3$$

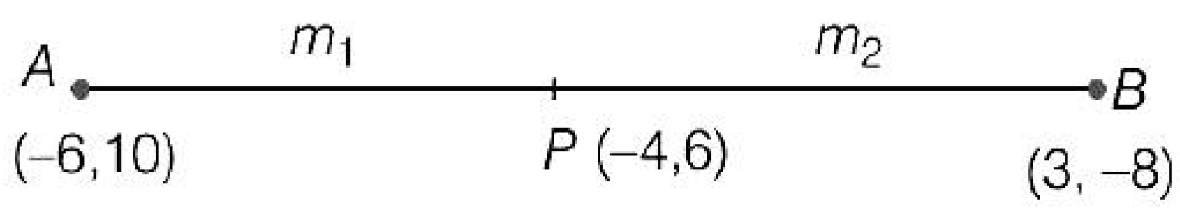
$$\Rightarrow \quad 36a - 27a - 8 - 1 = 0$$

$$\Rightarrow \quad 9a - 9 = 0$$

$$\therefore \quad a = 1$$

Hence, the required values of a and b are 1 and -3.

33. Let point $P(-4, 6)$ divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ in the ratio $m_1 : m_2$.



By using section formula, we get

$$(-4, 6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right) \quad \ldots\text{(i)}$$

On equating x-coordinate from both sides of Eq. (i), we get

$$-4 = \frac{3m_1 - 6m_2}{m_1 + m_2}$$

$$\Rightarrow \quad -4(m_1 + m_2) = 3m_1 - 6m_2$$

$$\Rightarrow \quad -4m_1 - 4m_2 = 3m_1 - 6m_2$$

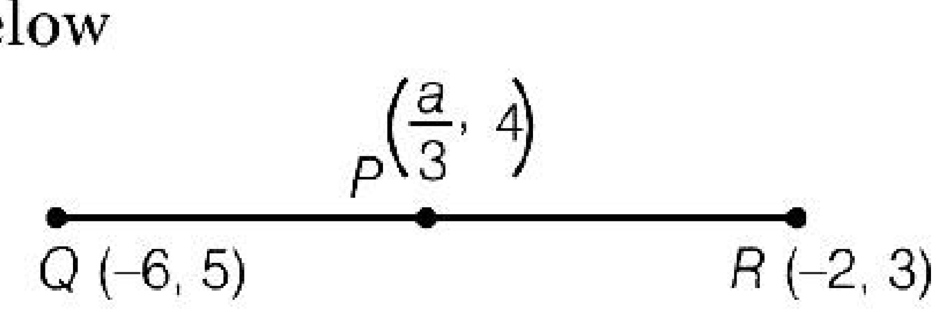
$$\Rightarrow \quad -4m_1 - 3m_1 = -6m_2 + 4m_2$$

$$\Rightarrow \quad -7m_1 = -2m_2$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{2}{7}$$

$$m_1 : m_2 = 2 : 7$$

34. Given that, $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, which shows in the figure given below



$$\therefore \text{ Mid-point of } QR = P\left(\frac{-6 - 2}{2}, \frac{5 + 3}{2}\right)$$

$$= P(-4, 4)$$

$\left[\text{since, mid-point of line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)\right]$

But mid-point $P\left(\frac{a}{3}, 4\right)$ is given.

$$\therefore \quad \left(\frac{a}{3}, 4\right) = (-4, 4)$$

On comparing the coordinates, we get

$$\frac{a}{3} = -4$$

$$\Rightarrow \quad a = -12$$

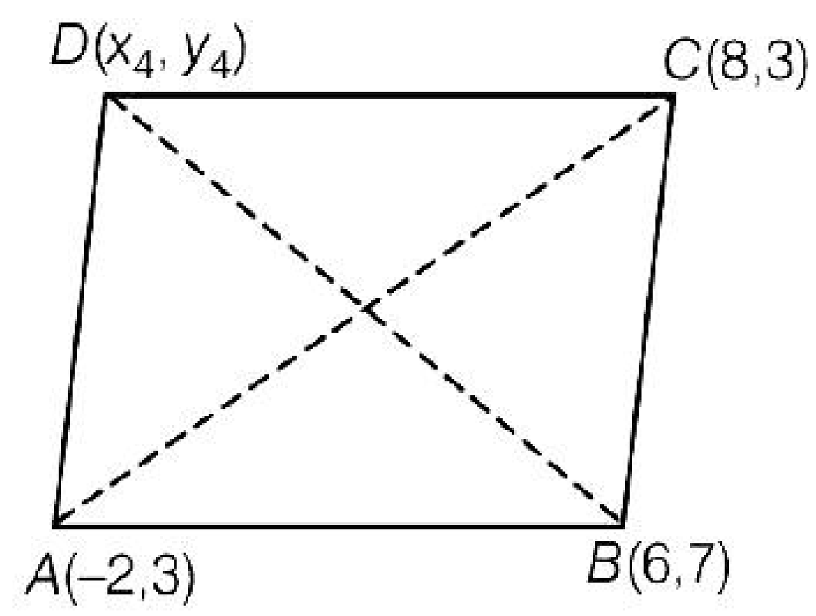
Hence, the required value of a is -12.

35. Let the fourth vertex of parallelogram, $D \equiv (x_4, y_4)$ and L, M be the middle points of AC and BD, respectively.

Then, $L \equiv \left(\frac{-2 + 8}{2}, \frac{3 + 3}{2}\right) \equiv (3, 3)$

$\left[\text{since, mid - point of a line segment having points } (x_1, y_1) \text{ and } = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$

and $\quad M \equiv \left(\frac{6 + x_4}{2}, \frac{7 + y_4}{2}\right)$



Since, $ABCD$ is a parallelogram, therefore diagonals AC and BD will bisect each other.

Hence, L and M are the same points.

$$\therefore \quad 3 = \frac{6 + x_4}{2} \text{ and } 3 = \frac{7 + y_4}{2}$$

$$\Rightarrow \quad 6 = 6 + x_4 \text{ and } 6 = 7 + y_4$$

$$\Rightarrow \quad x_4 = 0 \text{ and } y_4 = 6 - 7$$

$$\therefore \quad x_4 = 0 \text{ and } y_4 = -1$$

Hence, the fourth vertex of parallelogram is $D \equiv (x_4, y_4) \equiv (0, -1)$

36. Coordinate of the centroid G of ΔABC

$$= \left(\frac{-1+0-5}{3}, \frac{3+4+2}{3}\right) = (-2, 3)$$

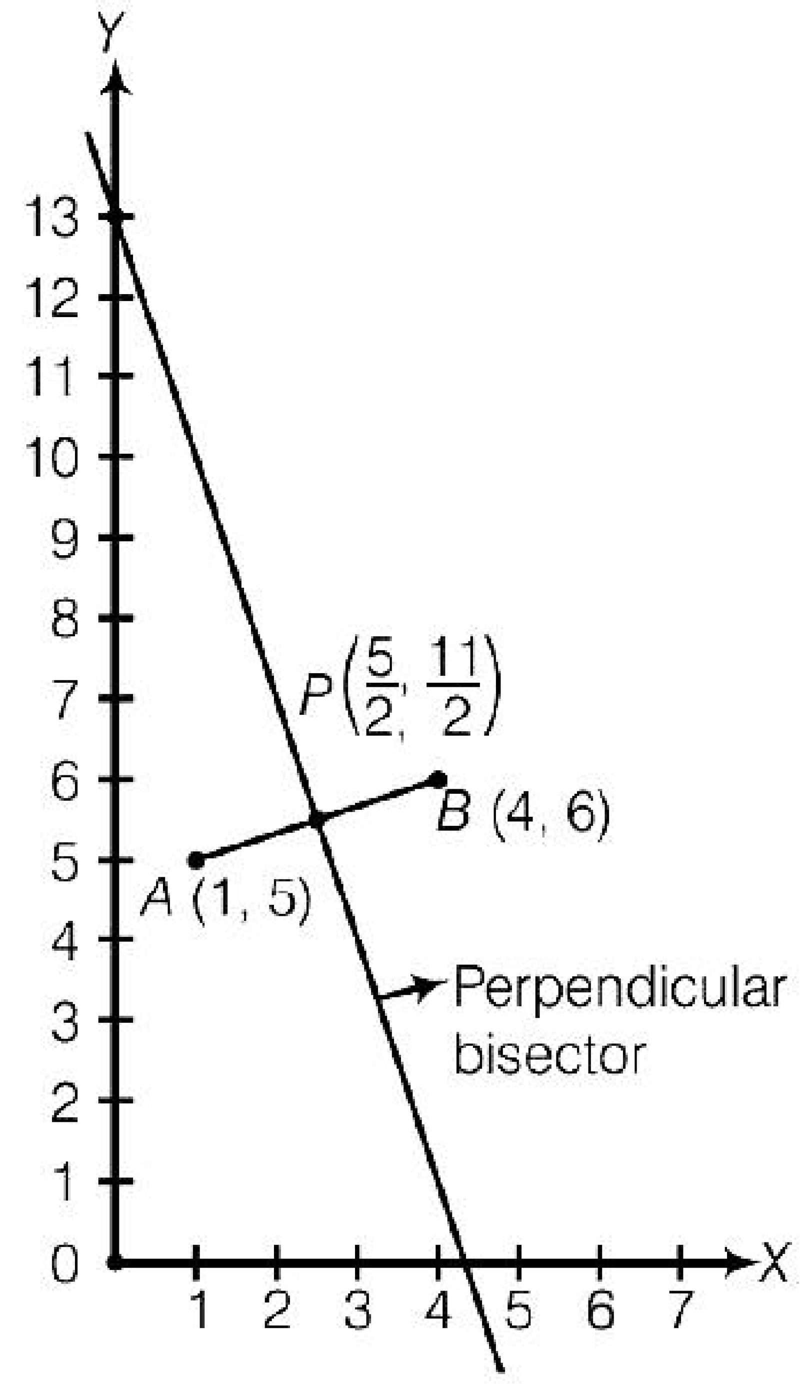
Since, G lies on the median $x - 2y + k = 0$.

So, G satisfy the equation $x - 2y + k = 0$.

$\therefore \quad -2 - 6 + k = 0$

$\Rightarrow \quad k = 8$

37. Firstly, we plot the points of the line segment on the paper and join them.



We know that, the perpendicular bisector of the line segment AB bisect the segment AB, i.e. perpendicular bisector of line segment AB passes through the mid-point of AB.

$\therefore$ Mid-point of $AB = \left(\frac{1+4}{2}, \frac{5+6}{2}\right)$

$\Rightarrow \quad P = \left(\frac{5}{2}, \frac{11}{2}\right)$

$\left[\because \text{mid-point of line segment passes through the points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

Now, we draw a straight line on paper passes through the mid-point P.

We see that the perpendicular bisector cuts the Y-axis at the point (0, 13).

Hence, the required point is (0, 13).

38. We know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

$\therefore$ Mid-point of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$

$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

$\left[\text{since, mid-point of any line segment which passes through the points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

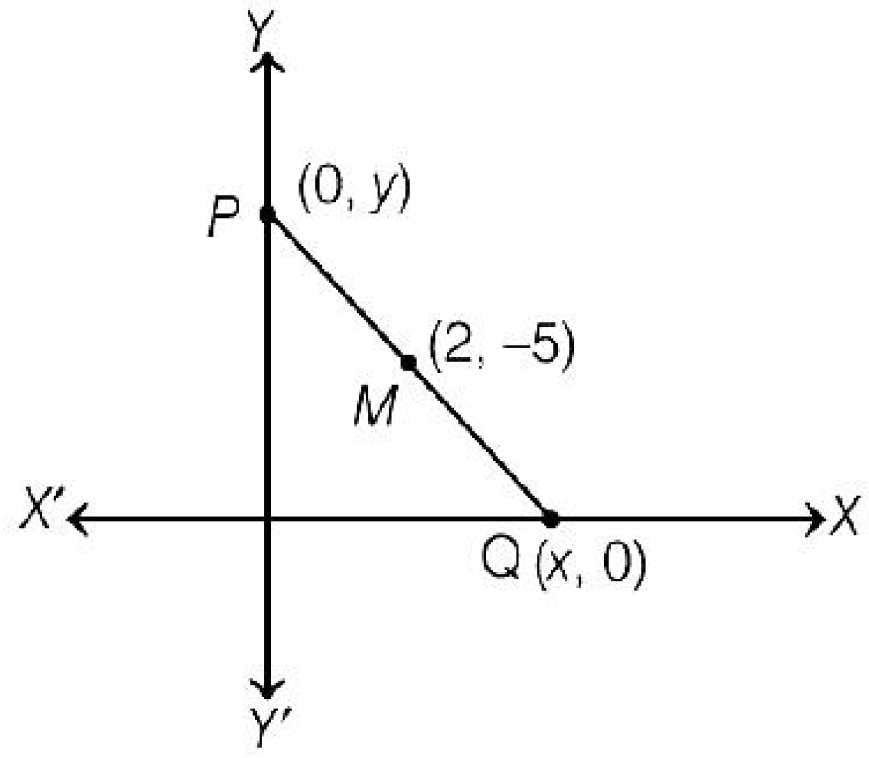
Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

39. Let the coordinates of P and Q (0, y) and (x, 0), respectively.

So, the mid-point of $P(0, y)$ and $Q(x, 0)$ is $M\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$.

$\left[\because \text{mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

But it is given that, mid-point of PQ is (2, –5).



$\therefore \quad 2 = \frac{x+0}{2}$ and $-5 = \frac{y+0}{2}$

$\Rightarrow \quad 4 = x$ and $-10 = y$

$\Rightarrow \quad x = 4$ and $y = -10$

So, the coordinates of P and Q are $(0, -10)$ and $(4, 0)$.

40. $\therefore$ Distance between $A(-2,0)$ and $B(2, 0)$,

$AB = \sqrt{[2-(-2)]^2 + (0-0)^2}$

$= \sqrt{(4)^2 + 0} = \sqrt{16} = 4$ units

[$\because$ distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$]

Similarly, distance between $B(2, 0)$ and $C(0, 2)$, $BC = \sqrt{(0-2)^2 + (2-0)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

In ΔABC, distance between $C(0, 2)$ and $A(-2, 0)$,

$CA = \sqrt{[0-(-2)]^2 + (2-0)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

Distance between $F(0, 4)$ and $D(-4, 0)$,

$FD = \sqrt{(0+4)^2 + (0-4)^2} = \sqrt{4^2 + (-4)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

Distance between $F(0, 4)$ and $E(4, 0)$,

$FE = \sqrt{(4-0)^2 + (0-4)^2} = \sqrt{4^2 + (-4)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

and distance between $E(4, 0)$ and $D(-4, 0)$,

$ED = \sqrt{[4-(-4)]^2 + (0)^2}$

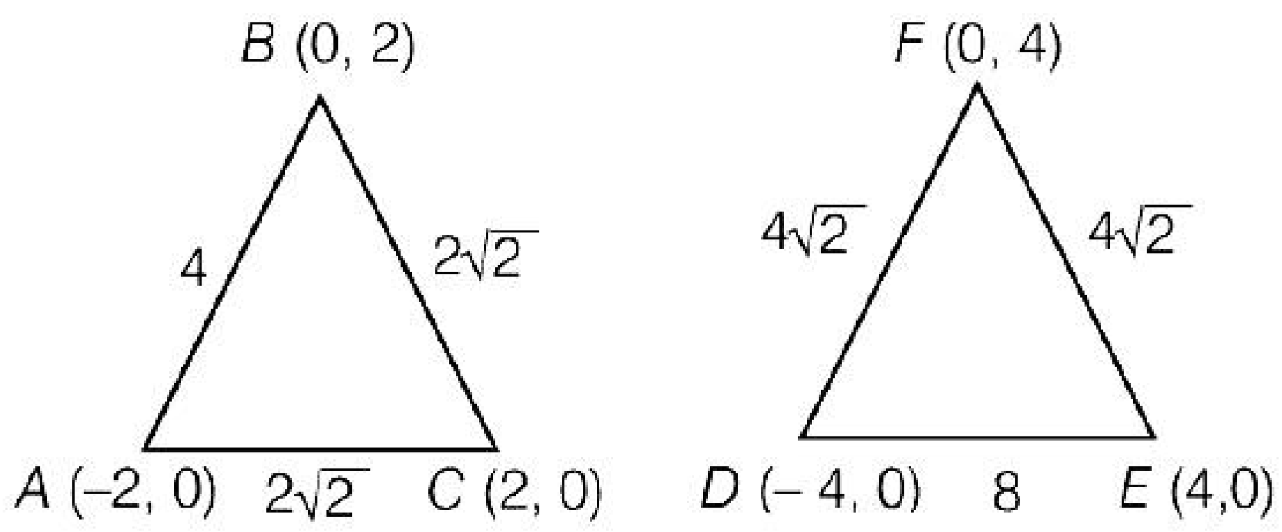
$= \sqrt{(4+4)^2 + 0} = \sqrt{8^2} = \sqrt{64} = 8$ units

Now, $\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}$, $\frac{AC}{DF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$,

$\frac{BC}{EF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$

$\therefore \quad \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Here, we see that sides of ΔABC and ΔFDE are propotional.



Hence, both the triangles are similar.

[by SSS rule]

41. Given, point (a, b) is the mid-point of the line segment joining the points $A(10, -6)$ and $B(k, 4)$.

$\therefore$ Coordinate of mid-point of

$$AB = \left(\frac{10+k}{2}, \frac{-6+4}{2}\right)$$

$$\Rightarrow \quad (a, b) = \left(\frac{10+k}{2}, -1\right)$$

Equate the x and y-coordinates both sides, we get

$$a = \frac{10+k}{2} \text{ and } b = -1$$

Also, given relation is

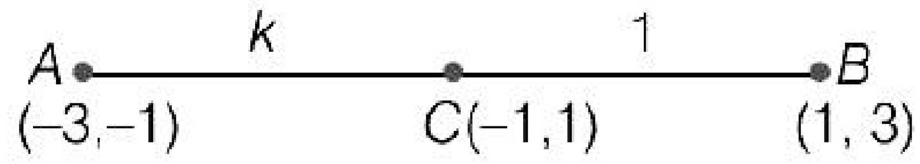
$$a - 2b = 18$$

$$\therefore \quad \frac{10+k}{2} - 2(-1) = 18$$

$$\Rightarrow \quad \frac{10+k}{2} = 18 - 2 \Rightarrow 10 + k = 2 \times 16$$

$$\Rightarrow \quad k = 32 - 10 \Rightarrow k = 22$$

42. Let $C(-1, 1)$ divides AB in the ratio $k:1$.



Then, by using section formula, we get

Coordinates of C are $\left(\frac{k-3}{k+1}, \frac{3k-1}{k+1}\right)$.

Thus, $C(-1, 1) = C\left(\frac{k-3}{k+1}, \frac{3k-1}{k+1}\right)$

On equating x-coordinate from both sides, we get

$$-1 = \frac{k-3}{k+1} \Rightarrow -k - 1 = k - 3$$

$$\Rightarrow \quad -2k = -3 + 1 \Rightarrow -2k = -2 \Rightarrow k = 1$$

On equating y-coordinate from both sides, we get

$$1=\frac{3k-1}{k+1}$$

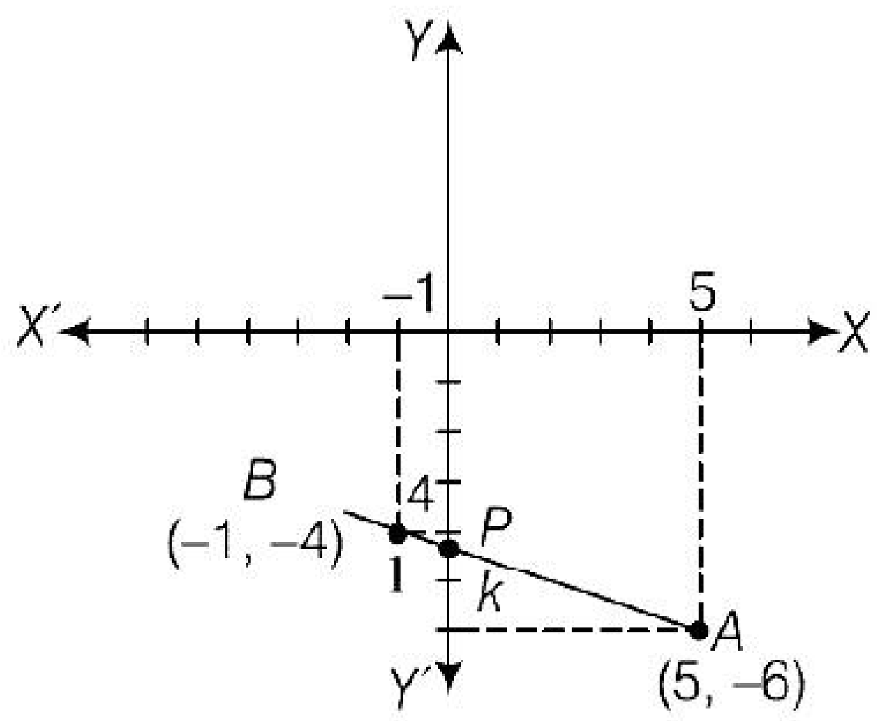
$\Rightarrow \quad k+1=3k-1$

$\Rightarrow \quad 2k=2 \Rightarrow k=1$

Since, in both cases value of k is same.
So, C divides AB in the ratio 1 :1, i.e. C is the mid-point of AB.

Hence, A, B and C are collinear.

43. Let AB be the given line segment with end points $A(5,-6)$ and $B(-1,-4)$. Also, let Y-axis divides AB in the ratio $k:1$ at point P.



Then, by using section formula,

$$\text{Coordinates of } P=\left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right) \quad \ldots\text{(i)}$$

Since, on the Y-axis, abscissa is 0.

So, x-coordinate of P will be zero.

On equating x-coordinate of P to 0, we get

$$\frac{-k+5}{k+1}=0$$

$\Rightarrow \quad -k+5=0 \Rightarrow k=5$

Thus, the required ratio is 5 : 1.

44. Let the coordinates of a point are (x, y).

We have, $\quad x_1=-1, y_1=7;$

$\quad x_2=4, y_2=-3$

and $\quad m_1=2, m_2=3$

$\therefore$ By using section formula,

$$x=\frac{m_1x_2+m_2x_1}{m_1+m_2}$$

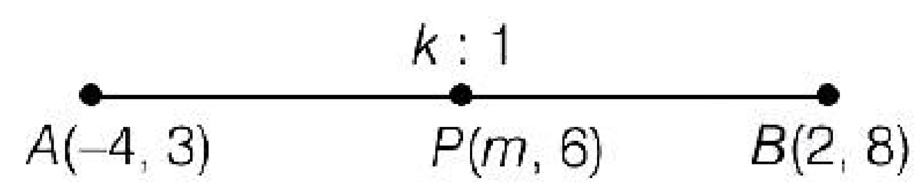
$$=\frac{2(4)+3(-1)}{2+3}=\frac{8-3}{5}=\frac{5}{5}=1$$

and $\quad y=\dfrac{m_1y_2+m_2y_1}{m_1+m_2}=\dfrac{2(-3)+3(7)}{2+3}$

$$=\frac{-6+21}{5}=\frac{15}{5}=3$$

Hence, coordinates of the point are $(1, 3)$.

45. $A(-4, 3)$, $B(2, 8)$ and $P(m, 6)$



Let P divides the join of AB in the ratio of $k:1$.

$$\therefore\ y\text{-coordinate of } P=\frac{k\times 8+1\times 3}{k+1}$$

$\Rightarrow \quad 6=\dfrac{8k+3}{k+1}$

$\Rightarrow \quad 6k+6=8k+3$

$\Rightarrow \quad 2k=3$

$\Rightarrow \quad k=\dfrac{3}{2}$

$\therefore$ P divides the join of AB in the ratio of 3 : 2.

46. Given, vertices of a parallelogram are $A(6, 1)$, $B(8, 2)$, $C(9, 4)$ and $D(p, 3)$.

Here, we have to find the value of p.

We know that, diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of mid-point of diagonal AC
= Coordinates of mid-point of diagonal BD

$$\Rightarrow \quad \left(\frac{6+9}{2}, \frac{1+4}{2}\right)=\left(\frac{8+p}{2}, \frac{2+3}{2}\right)$$

$$\left[\because \text{mid-point}=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

$$\Rightarrow \quad \left(\frac{15}{2}, \frac{5}{2}\right)=\left(\frac{8+p}{2}, \frac{5}{2}\right)$$

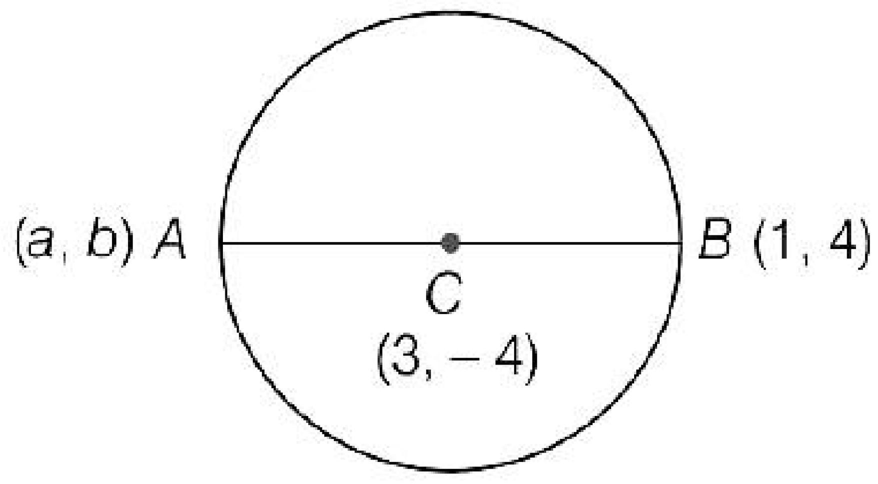
On equating x-coordinate from both sides, we get

$$\frac{15}{2}=\frac{8+p}{2} \Rightarrow 15=8+p$$

$\Rightarrow p=15-8 \Rightarrow p=7$

Hence, the required value of p is 7.

47. Let, AB be the diameter and C be the centre of the circle. Let coordinates of A be (a, b). Clearly, C will be the mid-point of AB.



$\therefore$ Coordinates of $C = \left(\dfrac{a+1}{2}, \dfrac{b+4}{2}\right)$

$$\left[\because \text{mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$$

$\Rightarrow \quad (3, -4) = \left(\dfrac{a+1}{2}, \dfrac{b+4}{2}\right)$

[given, coordinates of $C = (3, -4)$]

On comparing the coordinates of x and y from both sides, we get

$$\frac{a+1}{2} = 3 \quad \text{and} \quad \frac{b+4}{2} = -4$$

$\Rightarrow \quad a + 1 = 6 \quad$ and $\quad b + 4 = -8$

$\Rightarrow \quad a = 5$ and $b = -12$

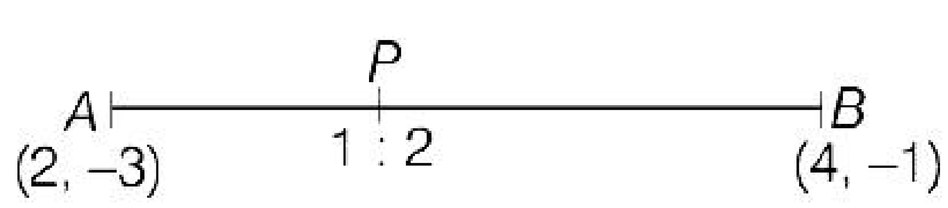
Hence, the coordinates of point A are $(5, -12)$.

48. Let P and Q be the points of trisection as shown below

Then, $AP : PB = 1 : 2$ and $AQ : QB = 2 : 1$

(i) When P divides AB in the ratio $1 : 2$.

Then, $\dfrac{m_1}{m_2} = \dfrac{1}{2}$



Here, $A(x_1, y_1) = (2, -3)$ and $B(x_2, y_2) = (4, -1)$

Now,

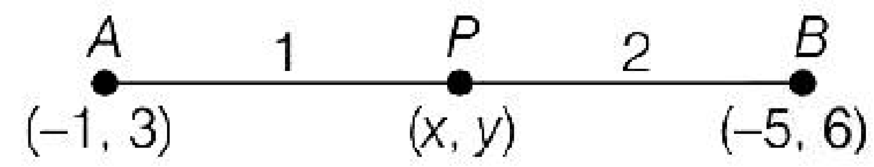
$$P(x, y) = P\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

[by section formula]

$$= P\left(\frac{1 \times 4 + 2 \times 2}{1+2}, \frac{1 \times (-1) + 2 \times (-3)}{1+2}\right)$$

$$= P\left(\frac{4+4}{3}, \frac{-1-6}{3}\right) = P\left(\frac{8}{3}, \frac{-7}{3}\right)$$

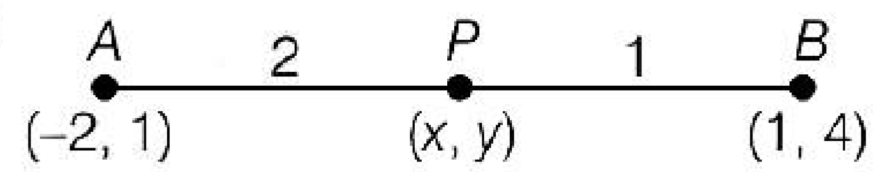
49. (P)



$$x = \frac{-5-2}{3} = -\frac{7}{3} \Rightarrow y = \frac{6+6}{3} = 4$$

$\therefore$ Point P is $\left(\dfrac{-7}{3}, 4\right)$

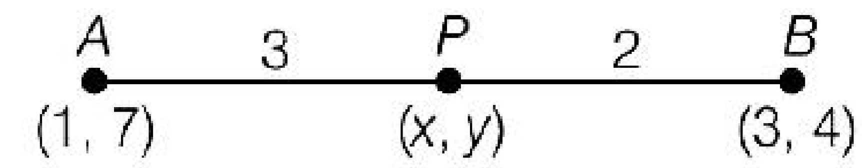
(Q)



$$x = \frac{2-2}{2+1} = 0, \; y = \frac{8+1}{2+1} = 3$$

$\therefore$ Point P is $(0, 3)$

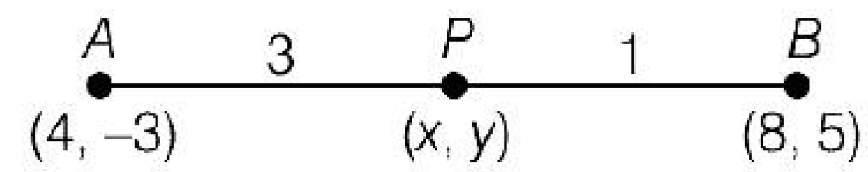
(R)



$$x = \frac{9+2}{5} = \frac{11}{5}, \; y = \frac{12+14}{5} = \frac{26}{5}$$

$\therefore$ Point P is $\left(\dfrac{11}{5}, \dfrac{26}{5}\right)$

(S)



$$x = \frac{24+4}{3+1} = 7, \; y = \frac{15-3}{3+1} = 3$$

$\therefore$ Point P is $(7, 3)$

50. (P) Distance between $(-6, 7)$ and $(-1, -5)$

$$= \sqrt{(-1+6)^2 + (-5-7)^2} = \sqrt{(5)^2 + 12)^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13 \text{ units}$$

(Q)

The given points are $A(k, -5)$ and $B(2, 7)$.

Now, $AB = 13 \Rightarrow AB^2 = 169$

$\Rightarrow (2-k)^2 + (7+5)^2 = 169$

$\Rightarrow k^2 - 4k + 4 + 144 = 169$

$\Rightarrow k^2 - 4k - 21 = 0$

$\Rightarrow (k-7)(k+3)=0$

$\Rightarrow k=7$ or $k=-3$

(R) $P(x, y)$, $A(5,1)$, $B(-1, 5)$

$$PA = PB$$

$$\Rightarrow \quad PA^2 = PB^2$$

$$\Rightarrow (5-x)^2+(1-y)^2=(-1-x)^2+(5-y)^2$$

$$\Rightarrow 25+x^2-10x+1+y^2-2y$$
$$=1+x^2+2x+25+y^2-10y$$

$$\Rightarrow \quad 3x=2y$$

(S) (x, y), $(2, 3)$ and $(4, 1)$ are collinear, then $x(3-1)+2(1-y)+4(y-3)=0$

$\Rightarrow 2x+2-2y+4y-12=0$

$\Rightarrow x+y=5$

51. Let a point on X-axis be $(x_1, 0)$, then its distance from the point $(2, 3)$

$$\Rightarrow \quad \sqrt{(x_1-2)^2+9}=c$$

$$\Rightarrow \quad (x_1-2)^2+9=c^2$$

$$\Rightarrow \quad x_1-2=\sqrt{c^2-9}$$

$\Rightarrow$ but $c<3 \Rightarrow c^2-9<0$

$\therefore x_1$ will be imaginary

Hence, both the Assertion are true and Reason is the correct explanation of Assertion.

52. The distance of point $P(x, y)$ from origin

$$=\sqrt{(x-0)^2+(y-0)^2}$$
$$=\sqrt{x^2+y^2}$$

Mid-point of $A(-3, b)$ and $B(1, b+4)$ is

$$\left(\frac{-3+1}{2}, \frac{b+b+4}{2}\right)=(-1, b+2)$$

$\therefore b+2=1 \Rightarrow b=-1$

Assertion is false but Reason is true.

53. Since, A and B lie on the circle having centre O.

$$OA = OB$$

$$\Rightarrow \sqrt{(4-2)^2+(3-3)^2}$$
$$=\sqrt{(x-2)^2+(5-3)^2}$$

$$\Rightarrow \quad 2=\sqrt{(x-2)^2+4}$$

$$\Rightarrow \quad 4=(x-2)^2+4$$

$$\Rightarrow \quad (x-2)^2=0 \Rightarrow x=2$$

Let the given points are $A(0, 0)$, $B(3, \sqrt{3})$ and $C(3, \lambda)$.

Since, ΔABC is an equilateral triangle, therefore

$$AB = AC$$

$$\Rightarrow \sqrt{(3-0)^2+(\sqrt{3}-0)^2}$$
$$=\sqrt{(3-0)^2+(\lambda-0)^2}$$

$$\Rightarrow \quad 9+3=9+\lambda^2 \Rightarrow \lambda^2=3$$

$$\Rightarrow \quad \lambda=\pm\sqrt{3}$$

Assertion is true but Reason is false.

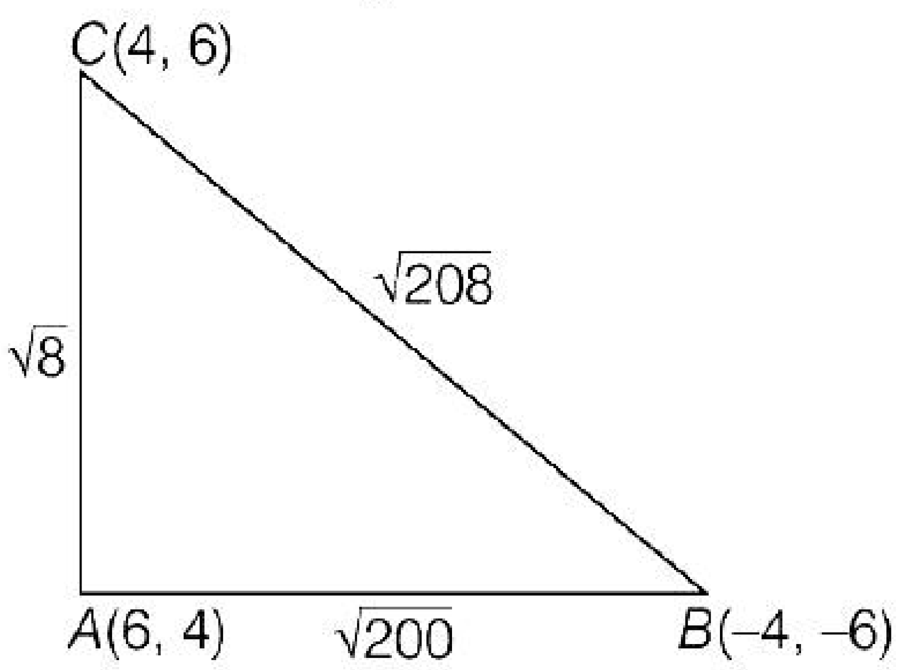
54. Three points A, B, C are collinear, if and onnly if $AB+BC=AC$ but here $AB+BC>AC$

$\therefore A, B, C$ are not collinear.

Hence, Assertion is false but Reason is true.

55. $\therefore AB+BC>AC$ and $AB+AC>BC$

$\therefore ABC$ is a triangle



[$\because$ Sum of the two sides is greater than third side]

Also, $BC^2=AB^2+AC^2$

$[\because (\sqrt{208})^2=(\sqrt{200})^2+(\sqrt{8})^2]$

$\therefore ABC$ is a right angled triangle.

Assertion is true Reason is true but is not the correct explanation of Assertion.

56. Let $A(3, 2)$, $B(-2, -3)$ and $C(2, 3)$.

$$\therefore AB=\sqrt{(-2-3)^2+(-3-2)^2}=\sqrt{50} \text{ units}$$

$$BC=\sqrt{(-2-2)^2+(-3-3)^2}=\sqrt{52} \text{ units}$$

and $CA = \sqrt{(2-3)^2 + (3-2)^2} = \sqrt{2}$ units

$\therefore\ BC^2 = AB^2 + CA^2$

$\Rightarrow \Delta ABC$ is a right triangle.

Let $A'(3, 6)$, $B'(-3, 4)$ and $P(x, y)$

Since, P is equidistant from A' and B', then

$$PA' = PB'$$

$$\Rightarrow \quad PA'^2 = PB'^2$$

$$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$\Rightarrow x^2 - 6x + 9 + y^2 - 12y + 36$$

$$= x^2 + 6x + 9 + y^2 - 8y + 16$$

$$\Rightarrow \quad 12x + 4y = 20$$

$$\Rightarrow \quad 3x + y = 5$$

Both the statements are true but the Reason is not correct explanation of the Assertion.

57. In quadrilateral $ABCD$

$AB = BC = CD = DA$ and $AC = BD$

Now will sides of quadrilateral are equal and diagonals $AC = BD$

$\therefore$ Quadrilateral is a square.

Assertion is true Reason is true and is the correct explanation of Assertion.

58. Here, $x_1 = 10\cos 30°$, $y_1 = 0$ and $x_2 = 0$, $y_2 = 10\cos 60°$

$\therefore$ Distance between the points

$$= \sqrt{(0 - 10\cos 30°)^2 + (10\cos 60° - 0)^2}$$

$$= \sqrt{\left(-10 \times \frac{\sqrt{3}}{2}\right)^2 + \left(10 \times \frac{1}{2}\right)^2}$$

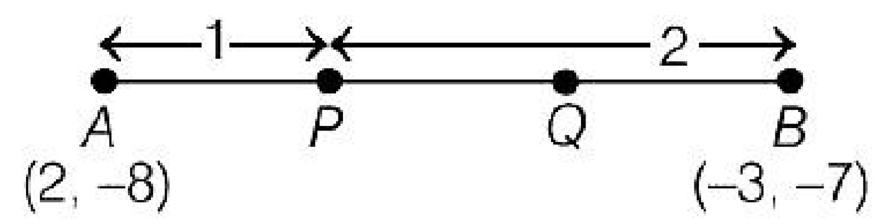
$$\left[\because \cos 30° = \frac{\sqrt{3}}{2} \text{ and } \cos 60° = \frac{1}{2}\right]$$

$$= \sqrt{\frac{300}{4} + \frac{100}{4}}$$

$$= \sqrt{\frac{400}{4}} = \sqrt{100} = 10 \text{ units}$$

Hence, Assertion is true but Reason is false as the mid-point is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

59.



$AP : PB = 1 : 2$

$\therefore$ Coordinates of $P = \left(\frac{-3+4}{3}, \frac{-7-16}{3}\right)$

$$= \left(\frac{1}{3}, \frac{-23}{3}\right)$$

Also, $AQ : QB = 2 : 1$

$\therefore$ Coordinates of $Q = \left(\frac{-6+2}{3}, \frac{-14-8}{3}\right)$

$$= \left(\frac{-4}{3}, \frac{-22}{3}\right)$$

Hence, Assertion is true but Reason is false

60. Both the statements are true but the Reason is not the correct explanation of the Assertion.

61. (i) In the figure, the coordinates of points are $P(4, 6)$, $Q(4, 2)$ and $R(8, 2)$.

Given, Malika's house is at $P(4, 6)$ and Karishma's house at $Q(4, 2)$

$\therefore$ Required distance $= PQ$

$$= \sqrt{(2-6)^2 + (4-4)^2}$$

$$= \sqrt{16 + 0} = 4 \text{ units}$$

(ii) Given, Karishma's house is at $Q(4, 2)$ and temple is at $R(8, 2)$

$\therefore$ Required distance $= QR$

$$= \sqrt{(2-2)^2 + (8-4)^2}$$

$$= \sqrt{0 + 16}$$

$$= 4 \text{ units}$$

(iii) Given, Malika's house is at $P(4, 6)$ and temple is at $R(8, 2)$

$\therefore$ Required distance $= PR$

$$= \sqrt{(2-6)^2 + (8-4)^2}$$

$$= \sqrt{16 + 16}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2} \text{ units}$$

(iv) As, $PQ = QR = 4$ units
and $PR = 4\sqrt{2}$ units
i.e., $PQ = QR \neq PR$
Hence, ΔPQR is an isosceles triangle.

(v) Given, School is at $O(0, 0)$ and Malika's house is at $P(4, 6)$

$\therefore$ Required distance $= OP$

$$= \sqrt{(6-0)^2 + (4-0)^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52} = 2\sqrt{13} \text{ units}$$

62. (i) As we have,

$$OP = 5 \text{ km}$$
$$\because \quad OP = \sqrt{(4)^2 + (y)^2}$$
$$25 = 16 + y^2 \qquad [\because OP = 5]$$
$$y^2 = 9$$
$$y = 3$$

(ii) As we have $PQ = 13$ km

$$PQ = \sqrt{(15-3)^2 + (x-4)^2}$$
$$169 = 144 + (x-4)^2$$
$$(x-4)^2 = 25$$
$$x - 4 = 5$$
$$x = 9$$

(iii) Given, Coordinates of P and Q are $P(4, 3)$ and $Q(9, 15)$

As, M is the mid point

$$\therefore \; M\left(\frac{4+9}{2}, \frac{3+15}{2}\right) = \left(\frac{13}{2}, \frac{18}{2}\right) = (6.5, 9)$$

(iv) Let P divides OQ in the ratio $K:1$

$$\therefore \quad 3 = \frac{K \times 15 + 1 \times 0}{K+1}$$
$$3K + 3 = 15K$$
$$12K = 3$$
$$K = \frac{1}{4} \quad \text{i.e., } 1:4$$

(v) Since, M is the mid-point of Noida and Delhi or $PM = MQ$

Hence, Anmol should try his luck moving towards Delhi.

63. (i) It can be observed that Niharika posted the green flag at $\frac{1}{4}$th distance of AD i.e. $\frac{1}{4} \times 100 = 25$ m from the starting point of 2nd line.

Therefore, the coordinates of this point G is (2. 25).

(ii) Preet posted red flag at $\frac{1}{5}$th distance of AD,

i.e. $\frac{1}{5} \times 100 = 20$ m from the starting point of 8th line.

Therefore, the coordinates of this point R is (8, 20).

(iii) Distance between these flags $= GR$

$$= \sqrt{(8-2)^2 + (25-20)^2}$$
$$= \sqrt{(6)^2 + (5)^2}$$
$$= \sqrt{36+25}$$
$$= \sqrt{61} \text{ m}$$

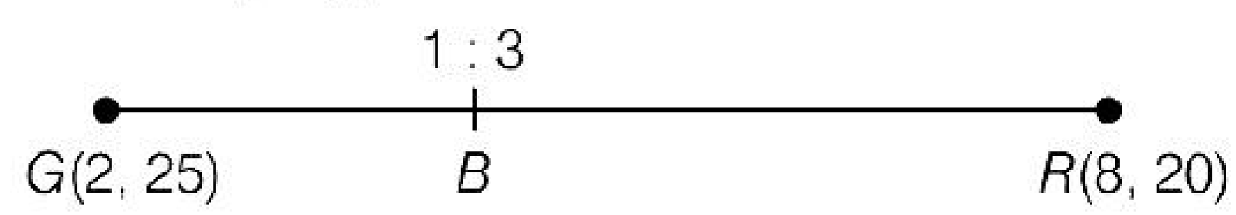
(iv) The point at which Rashmi should post her blue flag is the mid-point of the line joining these points.

Let this point be $A(x, y)$.

$$x = \frac{2+8}{2}, \; y = \frac{25+20}{2}$$
$$x = \frac{10}{2} = 5, \; y = \frac{45}{2} = 22.5$$

Hence, $A(x, y) = (5, 22.5)$

(v) Let the point at which Joy post his flag be $B(x, y)$.



Then, $x = \dfrac{1 \times 8 + 3 \times 2}{1 + 3}$,

$y = \dfrac{1 \times 20 + 3 \times 25}{1 + 3}$

$x = \dfrac{14}{4} = 3.5$,

$y = \dfrac{95}{4} = 23.75 \approx 24$

Hence, $B(x, y) = (3.5, 24)$

64. (i) Given, coordinates of $P(6, -2)$ and $R(10, 6)$

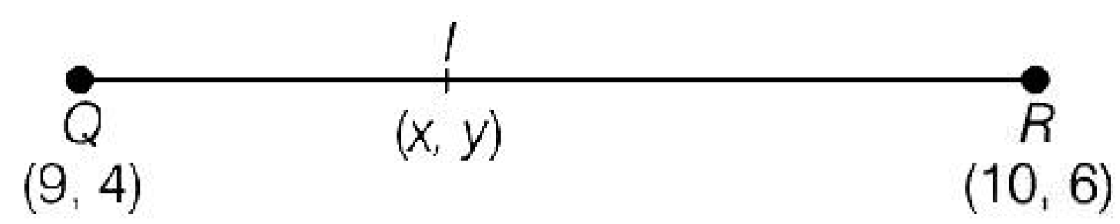
$\therefore$ Required distance

$= \sqrt{(6-(-2))^2 + (10-6)^2}$

$= \sqrt{64 + 16} = \sqrt{80}$

$= 4\sqrt{5}$ units

(ii) Let the coordinates of I be (x, y)



By section formula,

$x = \dfrac{1 \times 10 + 2 \times 9}{1 + 2} = \dfrac{10 + 18}{3} = \dfrac{28}{3}$

$y = \dfrac{1 \times 6 + 2 \times 4}{1 + 2} = \dfrac{6 + 8}{3} = \dfrac{14}{3}$

Hence, the coordinates of I is $\left(\dfrac{28}{3}, \dfrac{14}{3}\right)$

(iii) Given, coordinates of $P(6, -2)$ and $R(10, 6)$.

mid-point of

$PR = \left(\dfrac{6 + 10}{2}, \dfrac{-2 + 6}{2}\right)$

$= \left(\dfrac{16}{2}, \dfrac{4}{2}\right) = (8, 2)$

(iv) Let Q divides the segment PR in the ratio $K : 1$

Then, $9 = \dfrac{K \times 10 + 1 \times 6}{K + 1}$

$9 = \dfrac{10K + 6}{K + 1}$

$9K + 9 = 10K + 6$

$K = 3$

Hence, the ratio will be 3 : 1

(v) As, Q divides PR in the ratio 3 : 1

Hence, P, Q, R lies on a straight line.

65. (i) Here, $A(0, 6)$, $B(6, 6)$, $C(1, 1)$

$\therefore$ Coordinates of centroid of ΔABC

$= \left(\dfrac{0 + 6 + 1}{3}, \dfrac{6 + 6 + 1}{3}\right)$

$= \left(\dfrac{7}{3}, \dfrac{13}{3}\right)$

(ii) Given, Coordinates of A and B are $(0, 6)$ and $(6, 6)$.

Mid-point of $AB = \left(\dfrac{0 + 6}{2}, \dfrac{6 + 6}{2}\right)$

$= \left(\dfrac{6}{2}, \dfrac{12}{2}\right) = (3, 6)$

(iii) Given, Coordinates of B and C are $(6, 6)$ and $(1, 1)$

Mid-point of $BC = \left(\dfrac{6 + 1}{2}, \dfrac{6 + 1}{2}\right)$

$= \left(\dfrac{7}{2}, \dfrac{7}{2}\right)$

(iv) Given, Coordinates of A and C are $(0, 6)$ and $(1, 1)$

Mid-point of $AC = \left(\dfrac{0 + 1}{2}, \dfrac{6 + 1}{2}\right)$

$= \left(\dfrac{1}{2}, \dfrac{7}{2}\right)$

(v) Coordinates of S, T, and U are as $S(3, 6)$, $T\left(\dfrac{7}{2}, \dfrac{7}{2}\right)$ and $U\left(\dfrac{1}{2}, \dfrac{7}{2}\right)$

As we know, the centroid of triangle formed by joining the mid-points of sides of given triangle is same as that of the given triangle

Hence, centroid of $\Delta STU = \left(\dfrac{7}{3}, \dfrac{13}{3}\right)$